

Mechanics and Relativity: R1

September 30, 2022, Aletta Jacobshal

Duration: 90 mins

Before you start, read the following:

- There are 3 problems, for a total of xx points.
- Write your name and student number on all sheets.
- Make clear arguments and derivations and use correct notation. *Derive* means to start from first principles, and show all intermediate (mathematical) steps you used to get to your answer!
- Support your arguments by clear drawings where appropriate. If needed, draw your spacetime diagrams on the provided paper.
- Write your answers in the boxes provided. If you need more space, use the lined drafting paper.
- Generally use drafting paper for scratch work. Don't hand this in unless you ran out of space in the answer boxes.
- Write in a readable manner, illegible handwriting will not be graded.

	Points
Problem 1:	xx
Problem 2:	xx
Problem 3:	xx
Total:	xx
GRADE (1 + # Total/(9/xx))	

Useful equations:

$$\Delta s^2 = \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

$$\Delta t \geq \Delta s \geq \Delta \tau$$

Possibly relevant equations:

$$F = G \frac{Mm}{r^2}; \quad F = ma; \quad PV \propto k_b T; \quad F = \frac{dp}{dt}$$

Possibly relevant numbers:

$$c = 299792458 \text{ m/s} \quad (1)$$

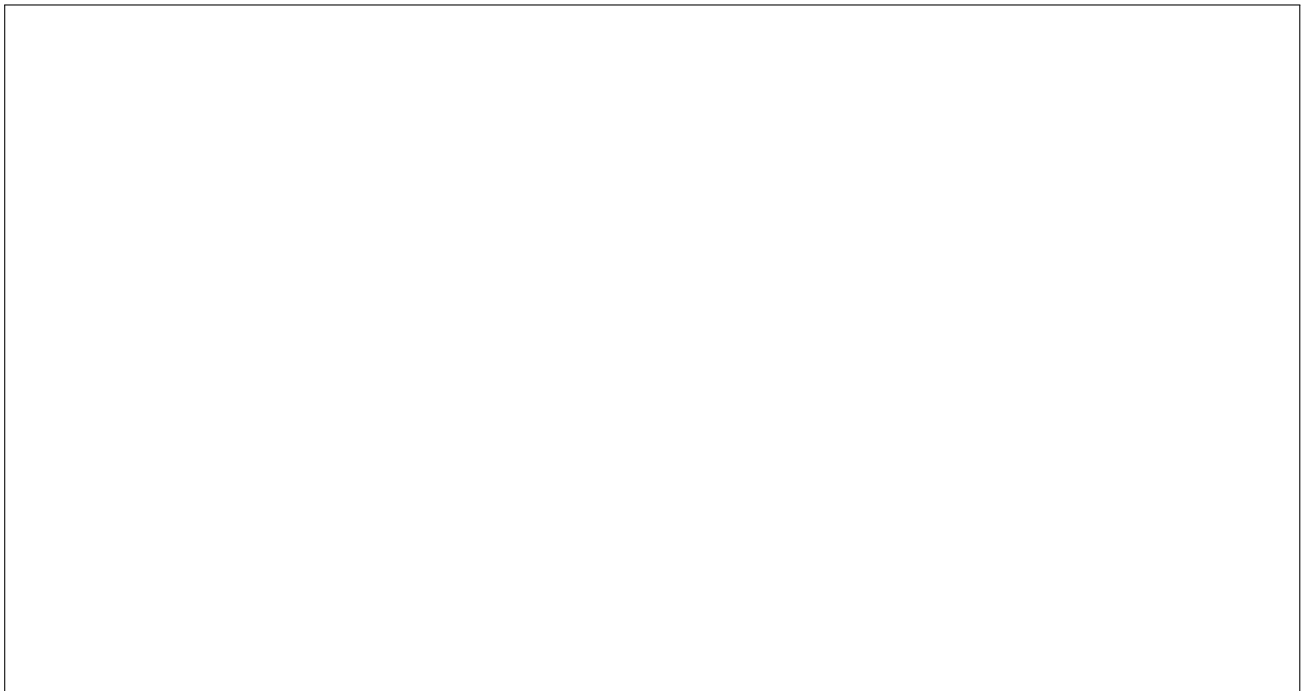
Question 1: Lightclock 2.0

Consider a lightclock as discussed during the lecture and in Moore. Now instead of deriving the spacetime interval, we aim to measure the time it takes for a ray of light (or a photon) to propagate from the source to the mirror and back.

- (a) **(4 pts)** Write down an expression for the time it takes for a ray of light to move from the source to the mirror and back. Assume the vertical length of the clock is given by L . Label this time as t_A .



- (b) **(4 pts)** Next, consider a frame in which the clock is moving to the right with velocity v . Sketch the trajectory of the light ray. Label the vertices of the light ray with the velocities v and c . **Possible Hint: recall that the speed of light is frame independent**



- (c) **(4 pts)** Use Pythagoras theorem to obtain the vertical velocity as observed in the frame where the clock is moving.

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- (d) **(1 pt)** Show that the time it takes for the light ray to move back and forth between the source and the mirror in the moving frame is given by $t_B = 2L/\sqrt{c^2 - v^2}$.

- (e) **(2 pts)** Compute the ratio between time t_B and t_A . Express your answer in terms of the so-called Lorentz factor $\gamma = 1/\sqrt{1 - (v^2/c^2)}$

- (f) **(4 pts)** The relation between t_A and t_B is known as time dilation. We will rederive this again in one of the lectures. Note btw that this is very similar to the relation we derived for proper time if we assume the velocity is constant, i.e. $\Delta\tau = \sqrt{1 - (v/c)^2}\Delta t = \gamma^{-1}\Delta t$. The reason is that we can consider a frame comoving¹ with the proper trajectory τ . In that case, the time elapsed in that frame $\Delta t' = \gamma^{-1}\Delta t$: time is dilated in the moving frame. Now consider $v = c$. What does this mean for time dilation? Explain your answer using the lightclock.

¹moving in the same direction and speed as

Question 2: Dimensional Analysis and Kepler's Laws

As you will learn in the Mechanics part of the course, Kepler's laws describe the elliptical orbit of a planet around the sun. His third law describes the relation between the period of the orbit T and the semi-major axis of the elliptical orbit a , this the largest distance between two points on the ellipse. You will derive the relation between T and a using dimensional analysis. The relation also involves the mass of the sun M and Newton's constant G .

Instruction: Use T, L and M for the *dimensions* of time, length and mass, respectively.

- (a) **(6 pts)** Explain why Kepler's third law depends on Newton's constant G and give its dimension.

- (b) **(12 pts)** Use dimensional analysis to *derive* the relation between T , a , G and M .²

- (c) **(2 pts)** In the relation you derived in above, only the mass of the sun M enters. Technically, this is only correct in case the mass of the planet m is much smaller than the sun's, $m \ll M$. How could the relation be adapted to account for the *additional* mass of the planet?

²Derive means to propose a power-law relationship between the variables and solve for the coefficients based on the required dimensions.

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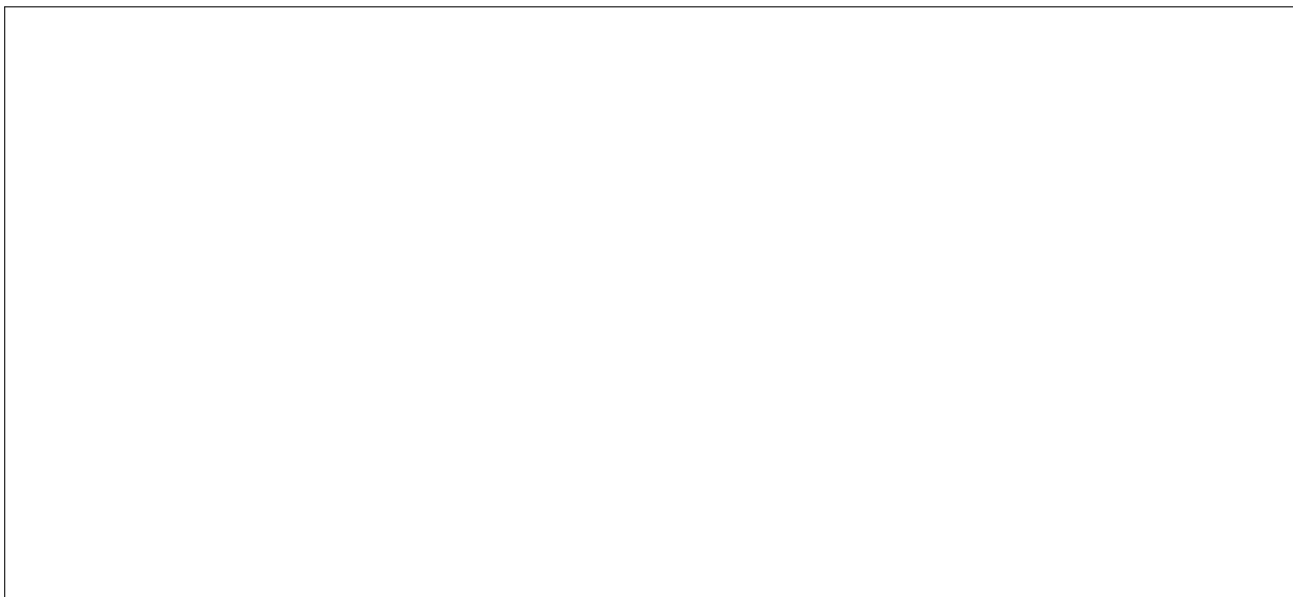
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Question 3: An Asteroid on the Way!

Dr. Kate Dibiascky discovers an asteroid at a constant speed of $0.8c$ that is on its way to Earth. Dr. Randall Mindy discovers it will wipe out most of life on Earth unless action is taken. From Earth, a radar signal is sent out to detect it (event A), which reflects from the asteroid (event B), and is detected (=seen) on Earth (event C). The coordinate time difference between events A and C is 6 hours. We want to find out when the asteroid hits Earth (D).

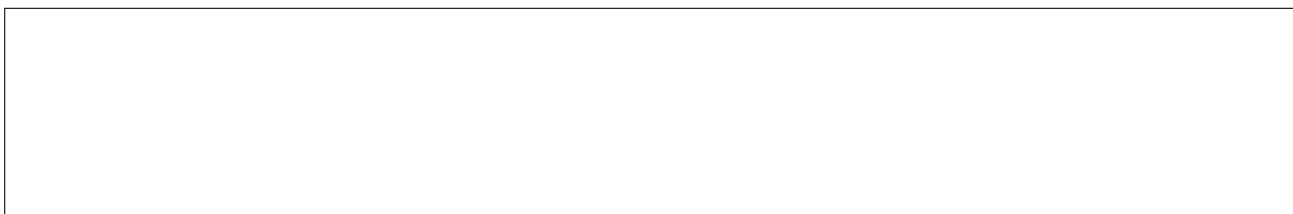
- (a) Draw a spacetime diagram. Draw and label the worldline of Earth and event B with respect to an observer at rest relative to the Earth. Make sure that the diagram can be read unambiguously by adding the necessary markings to the axes.



- (b) Using the diagram, find the time interval (Δt) between event C and the moment the asteroid strikes Earth (event D).



- (c) Write down an expression for the coordinates of event D in terms of the coordinates of event A and C.



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- (d) Write down an expression for the coordinates of event B in terms of the coordinates of event A and C.

- (e) Using the derived expression, check your answer in question (c).

- (f) What kind of time does each observer (consider an observer on Earth and an observer on the asteroid) measure between events B and D?

Half an hour after the signal is received with respect to an observer on Earth, Kate and Randall leave Earth at a constant speed of $0.6c$ in a spaceship (event E) and they move away from the asteroid (in the opposite direction to the asteroid).

- (g) Calculate the time measured between events E and D by the ship's clock explaining the formula you need to use. Hint: consider the discussion around 1(f), but realize who is measuring proper time.

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