Homework 1

All Things Stars - Solution Set

Problem 1: The Magnitude System

Question 1.
Explain how the scale of the magnitude system works and explain why the scale goes in the "wrong" direction compared to the conventional scales.

Solution 1.
The magnitude system is based on a logarithm of a ratio between fluxes of objects. The lower the magnitude, the bigger the ratio, and thus the brighter the object. The values can be negative for very bright objects.

Question 2.
Consider the following objects.

- The Sun
- The Moon, consider the full moon
- Andromeda Galaxy
- Betelgeuse
- Supernova 1987A, use the peak apparent magnitude

Answer the following questions with these objects.

a) Look up the apparent magnitudes in the V band of the objects and arrange them from the brightest to faintest. Include the value for magnitude you found.

b) What if they were all placed at 10 parsec away from us? Calculate the absolute magnitudes of the objects and arrange them again.

Solution 2.

a) We have the following correct order of objects:
The Sun > The Full Moon > Betelgeuse > Supernova 1987A > Andromeda Galaxy
These objects have the following magnitudes, in the same order as above:
-26.74 > -12.74 > 0.50 > 2.9 > 3.44

b) Students need to look up the distance to the object and use the correct units.

$$m - M = 5 \log\left(\frac{d}{10 \text{ pc}}\right) \quad \Leftrightarrow \quad M = m - 5 \log\left(\frac{d}{10\text{pc}}\right) \tag{1.1}$$

- **The Sun:** $d = 1.496 \cdot 10^{11} m, M = 4.83$
- **The Moon:** $d = 3.844 \cdot 10^{8} m, M = 31.78$
- **Andromeda:** $d = 765\text{kpc}, M = -20.98$
- **Betelgeuse:** $d = 168.1 \text{ pc}, M = 5.63$
- **Supernova 1987A:** $d = 51,400 \text{ pc}, M = -0.66$

This gives a new order of: Andromeda Galaxy > Supernova 1987A > The Sun > Betelgeuse > The Moon

Question 3.
At what distance would the Sun have to be in order for it to have the same apparent magnitude as a 100 Watt light bulb found 20 m away? Express your answer in meters, lightyears and parsec.

Solution 3.
Use the following relation for apparent magnitude:

$$m_2 - m_1 = 2.5 \log \left(\frac{F_1}{F_2} \right) \tag{1.2}$$

If the magnitudes are equal, the difference is zero. Thus the logarithm of fluxes has to be zero too. This is only possible if the ratio of fluxes is equal to one. Thus, the fluxes have to be equal. We have;

$$F_\odot = F_{LB} \qquad \text{and} \qquad F = \frac{L}{4\pi d^2}$$

We can look up the sun's luminosity; $L_\odot = 3.846 \cdot 10^{26}$ W, and we have the luminosity of the lightbulb, which is given in the question; $L_{LB} = 100$ W.

$$\frac{L_\odot}{4\pi d_\odot^2} = \frac{L_{LB}}{4\pi d_{LB}^2} \qquad \Rightarrow \qquad \frac{L_\odot}{d_\odot^2} = \frac{L_{LB}}{d_{LB}^2} \qquad \Rightarrow \qquad d_\odot = \sqrt{\frac{L_\odot}{L_{LB}} \cdot d_{LB}^2} = \sqrt{\frac{L_\odot}{L_{LB}}} \cdot d_{LB}$$

Filling in our variables we get that $d_\odot = \sqrt{\frac{3.846 \cdot 10^{26}\ \text{W}}{100\ \text{W}}} \cdot 20\ \text{m} = 3.92 \cdot 10^{13}$ m.

Converting this to the requested answers yield $d_\odot = 3.92 \cdot 10^{13}\ \text{m} = 4.14 \cdot 10^{-3}\ \text{ly} = 1.27 \cdot 10^{-3}$ pc.

Question 4.
A binary star system is observed, and since the separation between the two stars is much smaller than the distance of the system from the observer, it can be that both stars are found at the same distance from Earth. For the first star, the absolute magnitude is determined to be -1.0, while its apparent magnitude is 3. If the apparent magnitude of the second star is 5, what is its absolute magnitude? At what distance, in lightyears, is the binary system from the observer?

Solution 4.
There are two ways to solve this question.

Use the distance modulus to find the distance and then find the absolute magnitude.

We have:

$$m - M = 5 \log \left(\frac{d}{10\ \text{pc}} \right) \Rightarrow d = 10^{\frac{m-M}{5}+1}$$

Finding the distance yields

$$d = 10^{\frac{3.0-(-1.0)}{5}+1} = 10^{9/5} = 63.0957\ \text{pc} = 205.7906\ \text{ly}$$

Then finding the absolute magnitude for the second star:

$$M_2 = m_2 - 5 \log \left(\frac{63.0957}{10\ \text{pc}} \right) = 5 - 4 = 1$$

Use the fact that the stars are at the same distance.

We have:

$$m_1 - M_1 = 5 \log \left(\frac{d}{10\ \text{pc}} \right) = m_2 - M_2 \Rightarrow$$
$$m_1 - M_1 = m_2 - M_2$$

Getting the absolute magnitude, knowing the stars are at the same distance;

$$m_1 - M_1 = m_2 - M_2 \Rightarrow$$
$$3.0 - (-1.0) = 5 - M_2 \Rightarrow$$
$$4 - 5 = -M_2 \Rightarrow$$
$$M_2 = 1$$

Problem 2: Luminosity, Radius and Temperature

Question 1.
How could you estimate the temperature of a very hot object, e.g.; a metal rod, without touching it? Explain.

Solution 1.
You can estimate the temperature of an object by looking at its thermal emissions. If the rod is red we know it must be very hot, if it's glowing white we know it is extremely hot, etc.

Question 2.
What is the luminosity of a star A with 5 times the solar temperature and ½ the solar radius? Give your answer in solar luminosities ($L_\odot$).

Solution 2.
We have the following relation;

$$\frac{L}{L_\odot} = \left(\frac{T}{T_\odot}\right)^4 \left(\frac{R}{R_\odot}\right)^2 \tag{1.3}$$

We can substitute the variables that we already know about our star, yielding;

$$\frac{L}{L_\odot} = \left(\frac{5T_\odot}{T_\odot}\right)^4 \left(\frac{0.5R_\odot}{R_\odot}\right)^2 \Rightarrow \frac{L}{L_\odot} = (5)^4 \left(\frac{1}{2}\right)^2 \Rightarrow L = \frac{5^4}{2^2} L_\odot = \frac{625}{4} L_\odot \approx 156.25 L_\odot$$

Question 3.
Suppose a star M is twice as hot and three times as far as a star N, but they have the same apparent magnitude. What is the ratio of their radii, $\frac{R_2}{R_1}$?

Solution 3.
We start with the fact that the stars have the same magnitude and notice that in the following equation;

$$m_2 - m_1 = 2.5 \log \left(\frac{F_1}{F_2}\right) \tag{1.4}$$

If the magnitudes are equal, the logarithm of fluxes has to be zero. This happens when $F_1 = F_2$. We have;

$$F = \frac{L}{4\pi d^2} \quad \text{and} \quad L = \sigma_{SB} T^4 4\pi R^2 \quad \text{and the relation} \quad T_2 = 2T_1 \text{ and } d_2 = 3d_1$$

This results in a flux of $F = \frac{\sigma_{SB} T^4 4\pi R^2}{4\pi d^2} = \frac{\sigma_{SB} T^4 R^2}{d^2}$

Equating the fluxes as $F_1 = F_2$ yields;

$$\frac{\sigma_{SB} T_1^4 R_1^2}{d_1^2} = \frac{\sigma_{SB} T_2^4 R_2^2}{d_2^2} \Rightarrow \frac{T_1^4 R_1^2}{d_1^2} = \frac{T_2^4 R_2^2}{d_2^2} \Rightarrow \frac{R_2^2}{R_1^2} = \frac{T_1^4 d_2^2}{T_2^4 d_1^2} \Rightarrow \frac{R_2}{R_1} = \left(\frac{T_1}{T_2}\right)^2 \left(\frac{d_2}{d_1}\right)$$

Substituting what we know about the temperature and distance gives $\frac{R_2}{R_1} = \left(\frac{T_1}{2T_1}\right)^2 \left(\frac{3d_1}{d_1}\right) = \frac{3}{4}$

Question 4.
A red giant star has a temperature of $3500K$ and a luminosity $L = 300L_\odot$. Roughly, what is its radius in terms of solar radii $R_\odot$?

Solution 4.
Using relation (1.3) from question 2, we rearrange for the radius, we get;

$$\frac{R}{R_\odot} = \sqrt{\frac{L}{L_\odot}}\left(\frac{T_\odot}{T}\right)^2 \tag{1.5}$$

Using what we know about the red giant and the temperature of the sun $T_\odot \in \{5000 - 6000\}$, we get;

$$\frac{R}{R_\odot} = \sqrt{\frac{300L_\odot}{L_\odot}}\left(\frac{5772}{3500}\right)^2 \Rightarrow R = 47.1R_\odot$$

Problem 3: Distances

Question 1.
Answer the following questions regarding stellar parallax.

a) Explain the principles behind stellar parallax and how astronomers use this technique to measure astronomical distances.

b) What are the limitations of this technique?

Solution 1.
Stellar parallax is a measurement technique used by astronomers to determine the distances to stars in our galaxy. The basic principle relies on the apparent shift in the position of a nearby star against the background of more distant stars as the Earth orbits the sun. This shift is caused by the observers's change in perspective during some period.

Limitations include that parallax is most effective for relatively nearby stars, long periods of time are needed, high precision needed to measure small angles.

Question 2.
Imagine a highly unlikely scenario where Earth is at its perihelion and some other planet is at its aphelion and is in conjunction with Earth. You and the observer standing on the other planet both measure the parallax angle of a known star to be 0.23 arc-seconds. You find in your favorite astronomy book that the distance to this star is 37 lightyears. What planet is the observer measuring from?

Solution 2.
First, we find the distance between the planets from the parallax. We can use;

$$\frac{\alpha}{\text{arcsec}} = \frac{s/\text{AU}}{d/\text{ pc}} \tag{1.6}$$

Converting the distance to the star from lightyears to parsec, we get a distance of 11.34 pc. The separation of planets is then;

$$s[\text{AU}] = \alpha[\text{arcsec}] \cdot d[\text{ pc}] = 0.23 \cdot 11.34 = 2.6082\text{AU}$$

The perihelion of Earth is 0.983AU. Thus the aphelion of the other panets is $2.6082 = 0.983 = 1.6252$AU This is very close to 1.66AU, which is the aphelion for Mars.

Problem 4: Hertzsprung-Russell Diagram

Question 1.
Explain the significance of the HR diagram in astronomy. How and why is it used by astronomers?

Solution 1.
Good points include but are not limited to; classifying stars by their intrinsic properties, it plots color/temperature vs luminosity/magnitude, helps understand stellar evolution.

Question 2.
Make a rough sketch of a HR diagram and use it to answer the following questions.

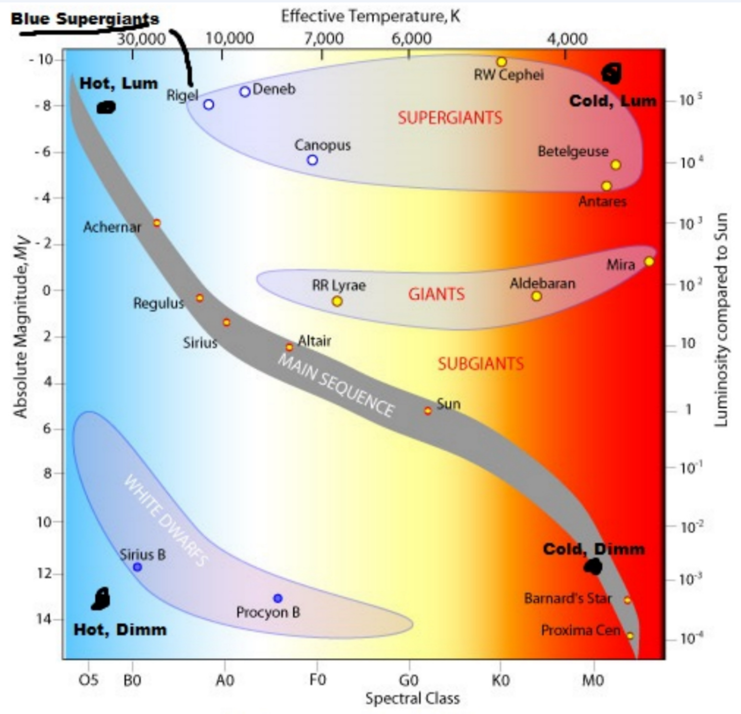
a) Indicate where you would find the following stars;

- Hot and luminous,
- Hot and dim,
- Cool and luminous,
- Cool and dim,
- The Sun.

b) Put labels on your diagram to mark;

- The Main Sequence,
- Red Giants,
- Blue Supergiants,
- Red Dwarfs.

Solution 2.
A solution is shown in the figure.



Problem 5: Stellar Parameters

Question 1.
Sirius A is a well known star amongst astronomy students at our University. It has a parallax of $p = 379$mas. Assume it is a spherical blackbody with radius $R = 1.711R_\odot$ and a surface temperature $T = 9940K$. Compute the following properties of Sirius A;

- Luminosity in erg/s and $L_\odot$.
- Absolute Magnitude.
- Apparent Magnitude.
- Distance Modulus.
- Peak wavelength $\lambda_{\max}$
- Indicate on the HR diagram from the previous questions where one would find Sirius A.

Solution 1.
For each of the points;

- For luminosity, we have; $L = \sigma_{SB}T^4 4\pi R^2 = 5.67037\times10^{-8}\cdot(9940)^4\cdot4\pi\cdot(1.71\cdot6.975\times10^8)^2 = 9.907\times10^{27}$ W

  We have that $L_\odot = 3.828 \times 10^{26}$ W, thus to express L in $L_\odot$ we get;

$$L = \frac{9.908 \times 10^{27}}{3.828 \times 10^{26}} = 25.88L_\odot$$

  We also have that $W = J \cdot s^{-1} \rightarrow J = kg \cdot m^2 \cdot s^{-2}, g = 10^{-3}kg, cm = 10^{-2}m \Rightarrow erg = 10^{-7}$ W, thus to express L in erg$/s$ we get;

$$L = 3.828 \times 10^{26} \cdot 10^7 \frac{erg/s}{\text{W}} = 3.828 \times 10^{33} \text{erg}/s$$

- For the absolute magnitude, we get;

$$M = 4.8 - 2.5\log\left(\frac{L}{L_\odot}\right) = 4.8 - 2.5\log(25.88) = 1.268$$

- For the apparent magnitude, we first calculate the distance to Sirius A using the information about the parallax. The student needs to realize that 379 mas = 379 $\times10^{-3}$ arcsecond, so;

$$\frac{\alpha}{\text{arcsec}} = \frac{s/\text{AU}}{d/\text{ pc}} \Rightarrow d = \frac{1\text{AU}}{379 \times 10^{-3}\text{arcsecond}} = 2.64 \text{ pc}$$

  We can then use;

$$m = 5\log\left(\frac{d}{10 \text{ pc}}\right) + M = 5\log\left(\frac{2.64}{10}\right) + 1.268 = -1.624$$

- To get the distance modulus, we use;

$$\mu = m - M = -1.624 - 1.268 = -2.892$$

- The peak wavelength $\lambda_{\max}$ can be calculated from;

$$\lambda_{\max} = \frac{290\text{nm}}{\frac{T}{10{,}000\text{K}}} = \frac{290}{\frac{9940}{10{,}000}} = \frac{290}{0.994} = 291.75\text{nm}$$

- For the location on the HR diagram, look at Problem 4: Solution 2.

Problem 6: Getting the Distance (*Last-years midterm question*)

Question 1.

a) You want to determine the distance to our neighboring disk galaxy M31 (the Andromeda Galaxy). Your method of choice would be:

(A) Parallaxes

(B) Cepheids

(C) Type Ia supernovae

(D) Hubble's Law

b) For **every** answer you have **not** chosen above, explain why this would not be the best method in 1 or 2 sentences.

Solution 1.
Question a
The correct answer is **B**

Question b
Parallaxes would be too close, Type Ia are quite rare, Hubble's law only works further out, actually M31 is moving towards us.

Cepheids are not perfect as well, because of internal extinction in the disk.