# Problem 1: The 21 cm line

The 21 cm line is an important transition line in Astronomy. It enables us to understand the distribution of neutral Hydrogen in the universe.

### Question 1.

Write the formula for the difference of two energy levels of an atom when emitting a photon. Write it in terms of  $\lambda$ ,  $E_0$  and  $E_1$ .

### Question 2.

Suppose the energy level of the lower state is  $E_0 = -13.6$  eV, find  $E_1$ .

# Question 3.

Compare this energy difference to the energy of a photon in the visible spectrum at 600 nm.

# Problem 2: Luminosity of a star

### Question 1.

Assuming that the sun can be approximated as a blackbody and that the temperature at the surface is 5000 degC, calculate the total luminosity of the sun using Stefan-Boltzmann law and the solar radius.

#### Question 2.

How many times more luminous is an O-class star with a surface temperature of  $30000 \, \mathrm{K}$  and a radius of  $10 \, \mathrm{solar}$  radii?

# Problem 3: Stellar Ages and Lifetimes

Before the discovery of thermonuclear reactions, it was commonly thought that gravity was the energy source of stars. If the gravitational energy of a given star of mass  $M_*$  and radius  $R_*$  is approximated by that of a body with a constant density, the energy emitted to space due to gravitational energy emitted since its formation may be approximated by the following expression;

$$\Delta U = \frac{3}{10} \frac{GM_*^2}{R_*} \tag{1.1}$$

### Question 1.

By assuming that the Sun's luminosity  $L_{\odot}$  has been constant since its formation, give a rough estimation of the age of the Sun assuming that gravity is the dominant source of energy for stars. This time is called the Kelvin - Helmholtz time.

Since the age of the Sun is much larger than its Kelvin-Helmholtz time, gravity is not sufficient to generate the enormous amount of energy irradiated by the Sun since its formation. Another possible energy source in stars is chemical energy.

### Question 2.

Assuming that 10 eV of energy per atom found in the Sun is emitted during some chemical reaction taking place there, calculate the time the Sun could shine at its present intensity if the only energy source was this chemical process. Assume that the Sun is composed of pure hydrogen.

# Problem 4: Binary Systems

# Question 1.

Two stars in a binary system are separated by an angular distance of 0.2" arcseconds. We would like to observe the sodium doublet, which is located at 589 nm for both stars. Use Rayleigh's criterion to determine how big the minimum diameter D of the primary mirror should be to distinguish both stars.

Note. Don't forget to convert from arcseconds to radians.

Another binary system is observed that consists of two stars orbiting a common centre of mass in a circular path. The orbital period is  $P_{\rm orb}=12$  days. The first star, observed in the visible band, has a measured orbital speed of  $v_1=34~{\rm km~s^{-1}}$  determined using a method called Doppler spectroscopy. The second star, an X-ray pulsar, orbits around the centre of mass at a radius of  $r_2=2.8\times 10^{10}~{\rm m}$ .

# Question 2.

What is the orbital radius of Star 1 about the centre of mass?

# Question 3.

What is the total orbital separation, i.e.; the distance, between Star 1 and Star 2?

# Question 4.

What is the total mass of the system?

### Question 5.

What are the respective masses of Star 1 and Star 2?

In some other cases of close binaries, the two stars actually pass in front of each other, forming an eclipse that temporarily reduces the amount of light we see. Two examples of light curves are given in Figure 1.1 and Figure 1.2.

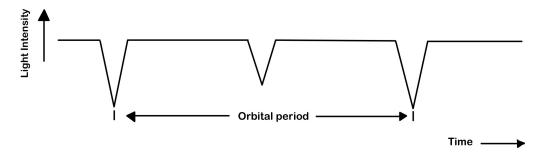


Figure 1.1

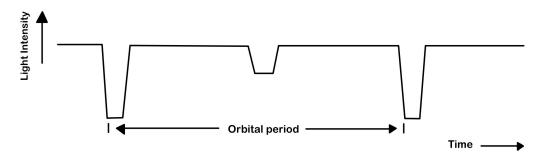


Figure 1.2

### Question 6.

What can you say about the relative radii of the binary stars if the observed lightcurve is similar to Figure 1.1 and if the lightcurve is similar to Figure 1.2?

### Question 7.

The net amount of stellar surface is during an eclipse the same whether the smaller or bigger star is in front. So why is one of the eclipses deeper than the other? What quantity determines which of the eclipses will be deeper?

# Problem 5: Night sky (from last-years midterm & exam)

# Question 1.

You want to observe a cluster with a telescope. Its coordinates are:  $12h\ 22.5m$  right ascension and +25 degrees declination. You use the Gratema telescope in Groningen at 53 degrees latitude. How many degrees do you have to tilt the telescope from the zenith to see the cluster?

- (A) 28 degrees
- (B) 62 degrees
- (C) That depends on the time of the day
- (D) That depends on the time of the year
- (E) C and D

# Question 2.

We are observing a binary system with the Gratema telescope from Groningen (53 degrees latitude). What is their maximum altitude above the horizon at any point in time during the year if their coordinates are RA: 06h34m 78s and DEC: 23d 00m 00s?

- (A) 30 degrees
- (B) 60 degrees
- (C) 90 degrees
- (D) They never get above the horizon

# Bonus problem: Stellarium

Use stellarium to answer the following questions:

### Question 1.

Place your observer in Groningen.

- a) Name three constellations that are circumpolar.
- b) What is the declination right above you, called Zenith? How does it change with your position on Earth?
- c) Go back to the year -12000. Is Polaris still the Pole Star? If not, which bright star is it? Explain this phenomena.

# Question 2.

Place your observer in Greenwich, United Kingdom.

- a) What is the Right Ascension and Declination of the Sun on the 21st of March at mid-day (12:00) 2023? What is this event called?
- b) What is the change in right ascension from the 21st to the 22nd of March at mid-day 2023? (23rd?, 24th?, etc...). Convert this to a time.
- c) How much will it change over an entire year? Try to explain this phenomena.