Problem 1: Evolution of the Sun

Question 1.
Calculate the lifetime of the Sun. Given that the mass of the Sun is $M_\odot = 2 \times 10^{30}$ kg and its luminosity is $L_\odot = 4 \times 10^{26}$ W. Assume the Sun consists of 72% hydrogen and all energy is produced through the ppI chain which releases a net energy of 26 MeV.

Solution 1.

$$0.72 \cdot \frac{2 \cdot 10^{30}}{1.67 \cdot 10^{-27}} = 8.6 \cdot 10^{56} \quad \text{protons}$$

4 protons create 1 helium + 26 MeV

$$26 \text{ MeV} \cdot 1.602 \cdot 10^{-19} \cdot 10^{6} = 4.17 \cdot 10^{-12} \text{ J}$$
$$\frac{4 \cdot 10^{26} \text{ W}}{4.17 \cdot 10^{-12} \text{ J}} \cdot 4 = 3.8 \cdot 10^{38} \text{ protons/s}$$
$$\frac{8.6 \cdot 10^{56}}{3.8 \cdot 10^{38}} = 2.24 \cdot 10^{18} \text{ s} = 7.1 \cdot 10^{10} \text{ years}$$

Question 2.
Look up the actual lifetime of the Sun. Which assumptions were made in the previous question, and how could the calculation be improved to become more accurate? (*describe, no calculation needed*)

Solution 2.
The actual lifetime of the Sun is around 10^{10} years. So our answer in the previous question is off by a factor 10. Firstly, we assumed all mass was converted into energy, when in reality less than 1% of the mass is. Secondly, nuclear fusion can only take place in the core of the Sun. When the Sun moves off the main sequence and becomes a red giant, only the hydrogen in the core finished burning. It is able to generate energy faster in the hydrogen-shell-burning phase.

Question 3.
After the Sun has exhausted its supply of hydrogen in the core, it will become a red giant. Describe this process and explain how hydrostatic equilibrium is regained. How do the properties of the Sun change when it becomes a red giant?

Solution 3.
In case of the sun, the hydrogen will have run out after 10 billion years. The core's temperature is not high enough for helium fusion to take place, therefore the fusion stops. This will cause the star's core to contract, causing it to heat up. This ignites the fusion of helium and hydrogen. The outer layers of the star expand, leading to a decrease in temperature as it is farther way from the star's core. This red giant will approximately be a hundred times larger than the current size of the sun and has a surface temperature of just 3500 degrees. The star will undergo several more processes while it will be moving around in the red giant region. During this time, it will develop distinct layers, eventually forming a carbon-hydrogen core.

Problem 2: Distances

One of the standard candles used to determine distances are cepheid variable stars. The observed light curve of cepheid is shown in Figure 1. Use the following period-luminosity relationship to determine the distance to this star.

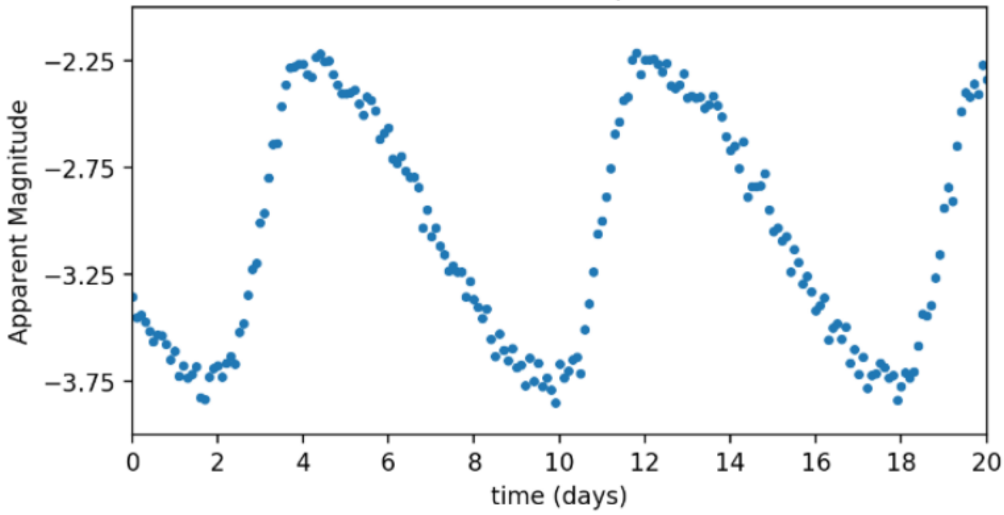

Figure 1: Light curve of a cepheid star

Question 1.
Together with Figure 3.2 use the following period-luminosity relationship to determine the distance to this star.

$$M = -2.78\log(P) - 1.35 \tag{1}$$

Where P is the period in days.

Solution 1.
Read of from graph: $m \approx 3.0$, and $P = 8$ days.

$$M = -2.78\log(8) - 1.35 = -3.86$$

distance modulus:

$$m - M = 5\log(d) - 5$$

$$D = 10^{\frac{m-M}{5}+1} = 14.9 \text{ pc}$$

Question 2.
What other method of distance measurement can be used to calibrate cepheid distances?

Solution 2.
see graph below

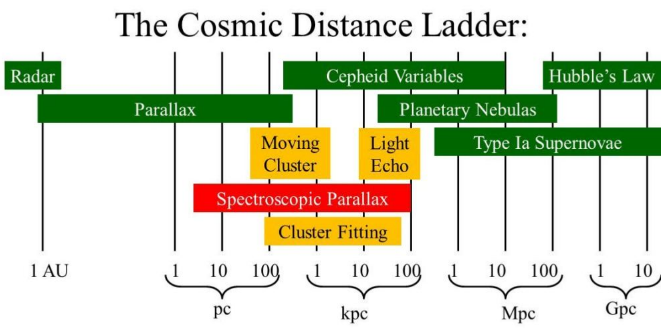

Figure 2: Cosmic distance ladder

Question 3.
In a spectrum from a distant galaxy, the Hα line (λ_{rest} = 656.3 nm) is observed at λ = 803.5 nm. Calculate the redshift and distance of this galaxy.

Solution 3.

$$z = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} = \frac{803.5 - 656.3}{656.3} = 0.22$$

$$v = c \cdot z = H \cdot d$$

$$d = \frac{cz}{H} = \frac{3 \cdot 10^5 \text{ km/s} \cdot 0.22}{70 \text{ km/s/Mpc}} = 943 \text{ Mpc}$$

Question 4.
How can dust in galaxies hinder the distance determination?

Solution 4.
Dust can cause reddening of wavelengths, which could make the redshift appear larger than it actually is, resulting in a strong overestimation of the distance. Dust dims the brightness of a galaxy, giving it a higher apparent magnitude compared to without dust attenuation.

Question 5.
Compare the cepheid variables and Hubble's law distance methods. For which distances and object types can each by applied?

Solution 5.
For cepheid variables, the cepheid stars needs to be resolved. Hence, this method is only applicable to cepheids in the Milky Way and nearby galaxies. Hubble's law is dependent on the expansion of the universe and therefore only works on more distant galaxies . For distance values see fig:cepheid.

Problem 3: Orbits

Question 1.
What is the gravitational acceleration due to the Sun at the location of the Earth's orbit, i.e.; at a distance of 1 AU?

Solution 1.
The acceleration is found by equating Newton's second law and the gravitational force

$$F = ma = \frac{GM_\odot m}{r^2} \tag{2}$$

The acceleration is therefore

$$a = \frac{GM_\odot}{r^2} = \frac{7 \times 10^{-11}\ \mathrm{N\ m^2/kg^2} \cdot 2 \times 10^{30}\ \mathrm{kg}}{\left(1.5 \times 10^{11}\ \mathrm{m}\right)^2} = 0.006\ \mathrm{m/s^2} \tag{3}$$

Question 2.
What would happen to the orbit of the Earth around the Sun and the value for the gravitational acceleration found in Q1) when the Sun happened to become a black hole?

Solution 2.
Nothing, all the properties, like the mass and the distance stay the same. Therefore, the gravitational acceleration does not change.

Question 3.
Orbiting around the black hole at the centre of the Milky Way, Star S0-19 has an orbital period of 37 years and a radius of 1720 AU. The orbit is actually not circular, but for now, use the expression derived for circular orbits to compute the mass of the black hole about which this star is orbiting. Express your answer in terms of solar masses, $M_\odot$.

Solution 3.
Kepler's third law is defined as

$$\frac{GM}{r^3} = \left(\frac{2\pi}{P}\right)^2 \tag{4}$$

This equation can be rewritten in order to obtain the Mass of the black hole

$$M \simeq \frac{4\pi^2 \left(1.44 \times 10^{14}\ \mathrm{m}\right)^3}{\left(7 \times 10^{-11}\ \mathrm{N - m^2/kg^2}\right)\left(4.73 \times 10^{8}\ \mathrm{s}\right)^2} \simeq 3.7 \times 10^6 M_\odot \tag{5}$$

Problem 4: Evidence for dark matter? *(last-year exam question)*

HI observations of an external spiral galaxy trace the gas disk out to 30 kpc from the center of the galaxy. At this radius, a rotation velocity of 280 km/sis observed.

Question 1.
What is the total mass inside that radius expressed in solar masses ?

Solution 1.

$$v_{orb} = \sqrt{\frac{GM}{R}} \Leftrightarrow M = \frac{v_{orb}^2 R}{G} \tag{6}$$

$$M = \frac{280^2 \cdot 30 \cdot 10^3}{4.3 \cdot 10^{-3}} = 5.5 \cdot 10^{11} M_\odot \tag{7}$$

Question 2.
You find that inside this radius the total luminosity of the galaxy (expressed in solar luminosities) is a factor 12 less than the total mass de-rived in question 1 (in solar masses). Would you take this as evidence for the existence of dark matter? Briefly explain why, or why not?

Solution 2.
Yes, stars alone cannot explain this mass as there would be more luminosity if they were responsible. We can conclude that there is something that we cannot see that is responsible for the extra mass.
Alternatively, it is possible that could be an alternate gravity theory, etc.

Question 3.
Can you describe in just a few lines one completely independent line of evidence supporting the existence of dark matter that we have dis-cussed during the course?

Solution 3.
From the Cosmic Microwave Background we see the fluctuations in density that were present shortly after the Big Bang. We can compare, with computer simulations, how these would grow into the structures we see in the Universe today and we need dark matter in order to bring both in agreement.

Problem 5: The Milky Way *(last-year exam question)*

Consider the following components of the Milky Way:

A) Thin disk
B) Thick disk
C) Bulge
D) Stellar halo

Question 4.
Make a sketch of the Milky Way seen edge-on (with the thin disk seen from the side) and indicate each of these components.

Solution 4.
Thick disk is drawn thicker and shorter than the thin disk. Bulge is in the middle, can contain a bar, doesn't have to, stellar halo is roughly spherical and goes out much farther than any other component.

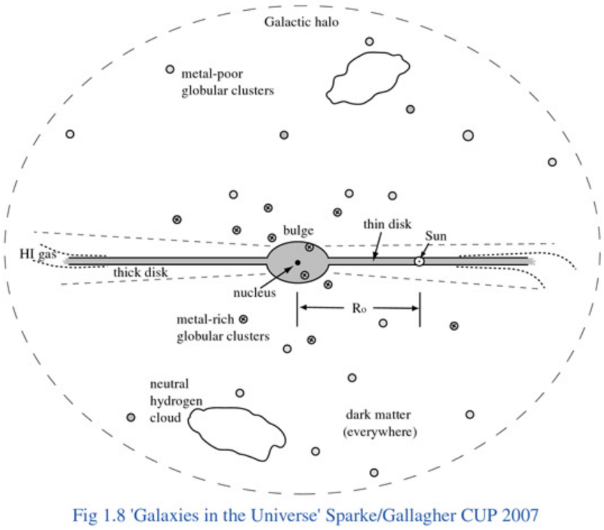

Figure 3: sketch of the Milky Way

Question 5.
Order all components A to D by average rotational velocity in the plane of the thin disk. Order from small to large rotational velocity.

Solution 5.
Thick disk rotates less fast than thin disk, halo has smallest rotation signature, thin disk has largest rotational signature, bulge is in between halo and thick disk.

Question 6.
Order all components A to D by velocity dispersion. Order from small to large velocity dispersion. Comment on what this means for the orbits of stars in the component you have given the largest velocity dispersion versus the component you have given the smallest velocity dispersion.

Solution 6.
Thick disk has larger velocity dispersion than thin disk, halo has highest, thin disk has smallest, bulge is in between halo and thick disk.
Comments: There is much less ordered motion in the halo than in the thin disk, the orbits are more random orientated, the orbits have a range of ellipticities, the Gaussian distribution of rotational motion has much larger wings.

Problem 6: Star formation - *Bonus question*

Question 1.
Stars form in gas clouds. Take a spherical gas cloud of pure hydrogen with a diameter of 10 pc and a density of $n(H_2) = 10^3$ cm^{-3}. What is the free fall velocity on the surface of this cloud?

Solution 1.

$$v_{ff} = \sqrt{\frac{2GM}{R}}$$

Where we first need to compute the mass :

$$M = \rho V = \frac{4}{3}\pi R^3 \rho$$

The density is given by:

$$\rho = n(H_2) \times m_{H_2}$$

unit conversion pc to cm, then fill back into the equation for v_{ff}

$$v_{ff} = \sqrt{2 \cdot 6.67 \cdot 10^{-8} \cdot 10^3 \cdot 2 \cdot 1.67 \cdot 10^{-24} \cdot \frac{4}{3}\pi(5 \cdot 3.086 \cdot 10^{18})^2}$$
$$\approx 6.6 \text{ km/s}$$

Question 2.
Stars typically form in groups. Why is this the case, and why are single stars very unlikely to form?

Solution 2.
Under its own gravity, a cloud begins to contract and fragment into parts that will become protostars. Young stars are found in open clusters and in loose associations, typically containing a few hundred stars which must have formed simultaneously.

Question 3.
Now consider a Galaxy containing 100 gas clouds ($n(H_2) = 10^3$ cm^{-3}), with an average mass of 5×10^4 $M_\odot$. What is the free fall timescale of such a cloud?

Solution 3.

$$t_{ff} = \sqrt{\frac{3\pi}{32G\rho}} = 3.64 \cdot 10^{13} \text{ s} = 1.2 \cdot 10^6 \text{ years}$$

Question 4.
For the same galaxy, estimate the star formation rate. Assume 10% of the mass is converted into stars.

Solution 4.

$$\text{Total mass SF} = \# \text{ clouds} \times \text{mass cloud} \times 10\%$$

$$\# \text{ stars} = \frac{\text{total mass}}{1 M_\odot}$$

$$\text{SFR} = \frac{\# \text{ stars}}{t_{ff}} \approx 0.43 \text{ M}_\odot/\text{yr}$$

Question 5.
Which different elements can be formed by a 1 $M_\odot$ and a 10 $M_\odot$ star throughout their lifetime. Which main physical processes are responsible for creating these elements?

Solution 5.

A 1 $M_\odot$ star can form helium (He) through hydrogen burning in the core. In the red giant phase, helium burning occurs, creating C and O. In the last stages during shell burning C, O, Ne, Mg, Si and S can be formed. A 10 $M_\odot$ star goes through more burning phases, creating elements up to Fe. Elements heavier than Fe are formed through neutron capture when the star undergoes a supernova explosion at the end of its life.