Homework 3

Distances & Stellar Evolution

Although the homeworks aren't graded, keep in mind that this is the only way to get comfortable with the way we ask questions on the exam.

Opportunity to ask questions about the questions in this homework set will be given in Tutorial 7.

Problem 1: Evolution of the Sun

Question 1.
Calculate the lifetime of the Sun, given that the mass of the Sun is $M_\odot = 2 \times 10^{30}$ kg and its luminosity is $L_\odot = 4 \times 10^{26}$ W. Assume the Sun consists of 72% hydrogen and all energy is produced through the ppI chain which releases a net energy of 26 MeV.

Question 2.
Look up the actual lifetime of the Sun. Which assumptions were made in the previous question, and how could the calculation be improved to become more accurate? (*describe, no calculation needed*)

Question 3.
After the Sun has exhausted its supply of hydrogen in the core, it will become a red giant. Describe this process and explain how hydrostatic equilibrium is regained. How do the properties of the Sun change when it becomes a red giant?

Problem 2: Distances

One of the standard candles used to determine distances are cepheid variable stars. The observed light curve of a Cepheid is shown in 3.1.

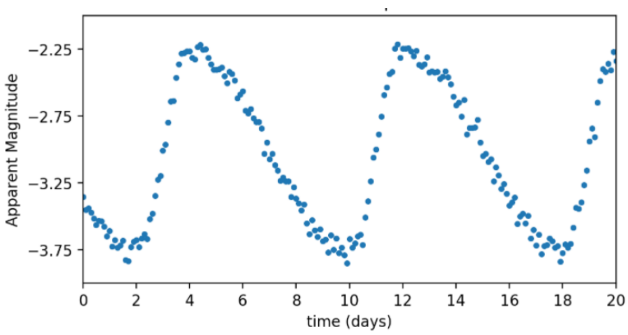

Figure 3.1: Light curve of a Cepheid star

Question 1.
Together with Figure 3.1, use the following period-luminosity relationship to determine the distance to this star.

$$M = -2.78 \log(P) - 1.35, \tag{3.1}$$

where P is the period in days.

Question 2.
What other method of distance measurement can be used to calibrate Cepheid distances?

Question 3.
In a spectrum from a distant galaxy, the Hα-line ($\lambda_{\text{rest}} = 656.3$ nm) is observed at $\lambda = 803.5$ nm. Calculate the redshift and distance of this galaxy.

Question 4.
How can dust in galaxies hinder the distance determination?

Question 5.
Compare the Cepheid variables and Hubble's law distance methods. For which distances and object types can each by applied?

Problem 3: Orbits

Question 1.
What is the gravitational acceleration due to the Sun at the location of the Earth's orbit, i.e.; at a distance of 1 AU?

Question 2.
What would happen to the orbit of the Earth around the Sun and the value for the gravitational acceleration found in Q1) when the Sun happened to become a black hole?

Question 3.
Orbiting around the black hole at the centre of the Milky Way, Star S0-19 has an orbital period of 37 years and a radius of 1720 AU. The orbit is actually not circular, but for now, use the expression derived for circular orbits to compute the mass of the black hole about which this star is orbiting. Express your answer in terms of solar masses, $M_\odot$.

Problem 4: Evidence for dark matter? *(last-year exam question)*

HI observations of an external spiral galaxy trace the gas disk out to 30 kpc from the center of the galaxy. At this radius, a rotation velocity of 280 km/s is observed.

Question 1.
What is the total mass inside that radius expressed in solar masses ?

Question 2.
You find that inside this radius the total luminosity of the galaxy (expressed in solar luminosities) is a factor 12 less than the total mass derived in question 1 (in solar masses). Would you take this as evidence for the existence of dark matter? Briefly explain why, or why not?

Question 3.
Can you describe in just a few lines one completely independent line of evidence supporting the existence of dark matter that we have discussed during the course?

Problem 5: The Milky Way *(last-year exam question)*

Consider the following components of the Milky Way:

A) Thin disk
B) Thick disk
C) Bulge
D) Stellar halo

Question 1.
Make a sketch of the Milky Way seen edge-on (with the thin disk seen from the side) and indicate each of these components.

Question 2.
Order all components A to D by average rotational velocity in the plane of the thin disk. Order from small to large rotational velocity.

Question 3.
Order all components A to D by velocity dispersion. Order from small to large velocity dispersion. Comment on what this means for the orbits of stars in the component you have given the largest velocity dispersion versus the component you have given the smallest velocity dispersion.

Problem 6: Star Formation - *Bonus problem*

Question 1.
Stars form in gas clouds. Take a spherical gas cloud of pure hydrogen with a diameter of 10 pc and a density of $n(H_2) = 10^3$ cm^{-3}. What is the free fall velocity on the surface of this cloud?

Question 2.
Stars typically form in groups. Why is this the case, and why are single stars very unlikely to form?

Question 3.
Now consider a Galaxy containing 100 gas clouds ($n(H_2) = 10^3$ cm^{-3}), with an average mass of $5 \times 10^4\ M_\odot$. What is the free fall timescale of such a cloud?

Question 4.
For the same galaxy, estimate the star formation rate. Assume 10% of the mass is converted into stars.

Question 5.
Which different elements can be formed by a 1 $M_\odot$ and a 10 $M_\odot$ star throughout their lifetime? Which main physical processes are responsible for creating these elements?