# Practice session: Introduction to Radio Astronomy

Kapteyn Learning Community Sirius A
Give motivations and/or derivations for your answers.

**This practice session is expected to be a bit easier than the final exam will be.**

1. **Question 1**
   Conceptual questions.
   (a) You perform an observation at 23 GHz. Why is it especially important to know about the weather conditions?
   (b) Describe the four contributions to the sky opacity between 1 GHz and 100 GHz.
   (c) Explain what brightness temperature is.
   (d) State the difference between flux and flux density.
   (e) State the Nyquist-Shannon sampling theory.
   (f) Explain why there is a low-frequency cut-off in the transmission of the atmosphere.
   (g) Write down an equation for the system temperature of the receiver, and explain each term.

2. **Question 2**
   Basic calculations
   (a) You know that the optical depth at zenith is 0.02. Assume that the density of the atmosphere can be approximated as AN exponential with a scale height of 8 kilometers. Calculate the opacity at sea level.
   (b) Starting from the equation of radiative transfer

$$\frac{dI}{ds} = -\kappa I + \epsilon, \tag{2}$$

   show that the brightness temperature is given by

$$T_b = T_s e^{-\tau} + T_{\text{atm}}(1 - e^{-\tau}) \tag{3}$$

   where $T_s$ is the source brightness temperature, $T_{\text{atm}}$ is the temperature of the atmosphere and $\tau$ is the optical depth of the atmosphere.
   (c) Some guy named Folkert wants to observe the galaxy Dwingeloo 1 with the 25-meter Dwingeloo Radiotelescope. Dwingeloo 1 is known to have a flux density at 21 cm of 48 mJy. The Dwingeloo Radiotelescope has an effective area of 314 $\text{m}^2$, a bandwidth of 10 MHz in the 21 cm band, and a system temperature of 36 K. Calculate the SEFD, and determine how long Folkert will have to integrate to get a 10-$\sigma$ detection.

3. **Question 3**
   Interferometry
   (a) Describe what an interferometer is and state 3 advantages they have over single-dish antennae.
   (b) The Very Large Array has 27 dishes. Calculate how much the sensitivity will improve by using all 27 dishes relative to a single dish.
   (c) Draw a basic diagram of a two-element interferometer including correlator, and indicate where the geometric delay comes from.
   (d) A silly 2nd-year student says it doesn't make any sense for an interferometer to have 27 dishes since your resolution is only determined by the longest baseline. Explain why it is necessary to have many dishes.

(e) Draw what the uv-coverage looks like for the Very Large Array for a single moment in time, and indicate what happens to the uv-coverage if you keep measuring for an hour and at multiple frequencies.

(f) You observe a perfect point source with an interferometer. What do you expect to see for the measured amplitudes as a function of baseline length?

(g) Explain what a dirty beam is and what it is used for.

4. **Question 4**
Emission mechanisms

(a) Show that the total power emitted by an accelerated charge $q$ is given by the following equation:

$$P = \frac{2}{3}\frac{q^2a^2}{c^3} \tag{14}$$

(b) Assuming you have a electron energy distribution of

$$N(E)dE = DE^{-\delta}dE \tag{15}$$

and given that you have an emission coefficient of

$$\epsilon_\nu d\nu = -\frac{dE}{dt}N(E)dE \tag{16}$$

show that the resulting spectrum is of the form

$$S_\nu \propto \nu^\alpha. \tag{17}$$

You may use the approximation that electrons radiate away most of their energy at a frequency given by $\nu = \gamma^2\nu_G$, and you may use the power emitted by a single relativistic electron: $P = \frac{4}{3}\sigma_T\beta^2\gamma^2cU_B$.

5. **Question 5**
Fourier fun times! :D

(a) Perform a Fourier transform of a Dirac delta function: $f(x) = A\delta_{\mathrm{D}}(x-d)$.

(b) How can this result be interpreted in terms of observing a point source with a 2-element interferometer?

(c) What happens when you change the location/amplitude of the point source with respect to the telescope? Interpret your result.

(d) Perform a Fourier transform of a top hat function of width D and height 1.

(e) How can this Fourier transform be generalized for a rectangular source?

(f) Using a drawing of the sinc function explain why it can be difficult to observe faint sources next to bright sources.