



Practice session: Introduction to Radio Astronomy

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Give motivations and/or derivations for your answers.

This practice session is expected to be a bit easier than the final exam will be.

1. Question 1

Conceptual questions.

- You perform an observation at 23 GHz. Why is it especially important to know about the weather conditions?
- Describe the four contributions to the sky opacity between 1 GHz and 100 GHz.
- Explain what brightness temperature is.
- State the difference between flux and flux density.
- State the Nyquist-Shannon sampling theory.
- Explain why there is a low-frequency cut-off in the transmission of the atmosphere.
- Write down an equation for the system temperature of the receiver, and explain each term.

Solution:

- The frequency of 23 GHz is very close to the broad absorption line due to water vapor in the atmosphere (~ 22.2 GHz). If you know the humidity of the atmosphere, you can correct for this absorption.
- Broadband opacity, which is opacity of dry air. This is practically independent of frequency.
 - Molecular absorption, which is absorption mostly due to molecular oxygen in the atmosphere. This contribution is dominant between 52 GHz and 60 GHz.
 - Hydrosols, which basically are water droplets like you find in clouds. This causes absorption proportional to λ^{-2} .
 - Water vapour, which is water in a gaseous state. This peaks at 22.2 GHz, and has a width of 4 GHz.
- The brightness temperature is the temperature a blackbody would have to give the same intensity at a specific frequency.
- Flux density is the flux integrated over a region on the sky.
- The Nyquist-Shannon sampling theorem states that you need a sampling frequency of 2ν to sample a signal with a frequency of ν .
- Below a certain wavelength, the atmosphere starts to reflect all emission. This threshold is determined by the concentration of charged particles in the atmosphere.
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$$T_{\text{sys}} = T_{\text{source}} + T_{\text{cmb}} + T_{\text{atm}} + T_{\text{spill}} + T_{\text{loss}} + T_{\text{cal}} + T_{\text{rx}} \quad (1)$$

T_{source} is the brightness temperature from the observed source. T_{cmb} is the temperature of the Cosmic Microwave Background. T_{atm} is brightness temperature of the atmosphere. T_{spill} is brightness temperature due to ground emission making it into the receiver. T_{loss} is the noise from losses in the receiver. T_{cal} is the noise temperature due to calibration. Finally, T_{rx} is the temperature of the receiver itself.



2. Question 2

Basic calculations

- (a) You know that the optical depth at zenith is 0.02. Assume that the density of the atmosphere can be approximated as AN exponential with a scale height of 8 kilometers. Calculate the opacity at sea level.
- (b) Starting from the equation of radiative transfer

$$\frac{dI}{ds} = -\kappa I + \epsilon, \quad (2)$$

show that the brightness temperature is given by

$$T_b = T_s e^{-\tau} + T_{\text{atm}}(1 - e^{-\tau}) \quad (3)$$

where T_s is the source brightness temperature, T_{atm} is the temperature of the atmosphere and τ is the optical depth of the atmosphere.

- (c) Some guy named Folkert wants to observe the galaxy Dwingeloo 1 with the 25-meter Dwingeloo Radiotelescope. Dwingeloo 1 is known to have a flux density at 21 cm of 48 mJy. The Dwingeloo Radiotelescope has an effective area of 314 m², a bandwidth of 10 MHz in the 21 cm band, and a system temperature of 36 K. Calculate the SEFD, and determine how long Folkert will have to integrate to get a 10- σ detection.

Solution:

- (a) Since opacity is proportional with density, we can write that

$$0.02 = \int_0^{\infty} \kappa_0 \cdot \exp(-s/8 \text{ km}) ds \quad (4)$$

where κ_0 is the opacity at sea level. Solving for κ_0 gives a value of $2.5 \cdot 10^{-8} \text{ cm}^{-1}$.

- (b) First, we can divide both sides of the equation by κ . We then have to remember that $d\tau = -\kappa ds$ (negative sign because ds is measured from the top of the atmosphere, while we will integrate from the ground up) and that the emission term in the radiative transfer equation is due to blackbody radiation from the atmosphere. We then have

$$\frac{dI}{d\tau} = I - B(T_{\text{atm}}). \quad (5)$$

Now multiply with $e^{-\tau}$ and integrate over the complete atmosphere:

$$\int_0^{\tau} \frac{dI}{d\tau'} e^{-\tau'} d\tau' = \int_0^{\tau} [I - B(T_{\text{atm}})] e^{-\tau'} d\tau'. \quad (6)$$

Using integration by parts, we then find

$$e^{-\tau'} I \Big|_0^{\tau} - \int_0^{\tau} -e^{-\tau'} I d\tau' = \int_0^{\tau} e^{-\tau'} I d\tau' - B(T_{\text{atm}}) \int_0^{\tau} e^{-\tau'} d\tau'. \quad (7)$$

The second integral on the left-hand side cancels with the first integral on the right-hand side. Next, we write everything out. This gives

$$I_s e^{-\tau} - I_{\text{obs}} = B(T_{\text{atm}})(e^{-\tau} - 1) \quad (8)$$

where I_s is the source intensity. Rearranging this gives

$$I_{\text{obs}} = I_s e^{-\tau} + B(T_{\text{atm}})(1 - e^{-\tau}). \quad (9)$$

Using the definition of brightness temperature, this can easily be rewritten to

$$T_b = T_s e^{-\tau} + T_{\text{atm}}(1 - e^{-\tau}). \quad (10)$$



- (c) The SEFD can be calculated as

$$\text{SEFD} = \frac{2k_B T_{\text{sys}}}{A_{\text{eff}}}. \quad (11)$$

Filling in the proper values gives an SEFD of 316 Jy. Assuming you want a $10\text{-}\sigma$ detection, you need the rms uncertainty to be a tenth of 48 mJy, which is 4.8 mJy. If we rewrite the equation for the rms uncertainty to an equation for the integration time, we find

$$t_{\text{int}} = \frac{1}{\Delta\nu} \left(\frac{\text{SEFD}}{\sigma_{S_\nu}} \right)^2. \quad (12)$$

Filling in the proper values results in an integration time of 6.5 minutes. In reality Folkert will have to integrate much longer because Dwingeloo 1 lies in the plane of the Milky Way, which means there is a lot of extra background noise.

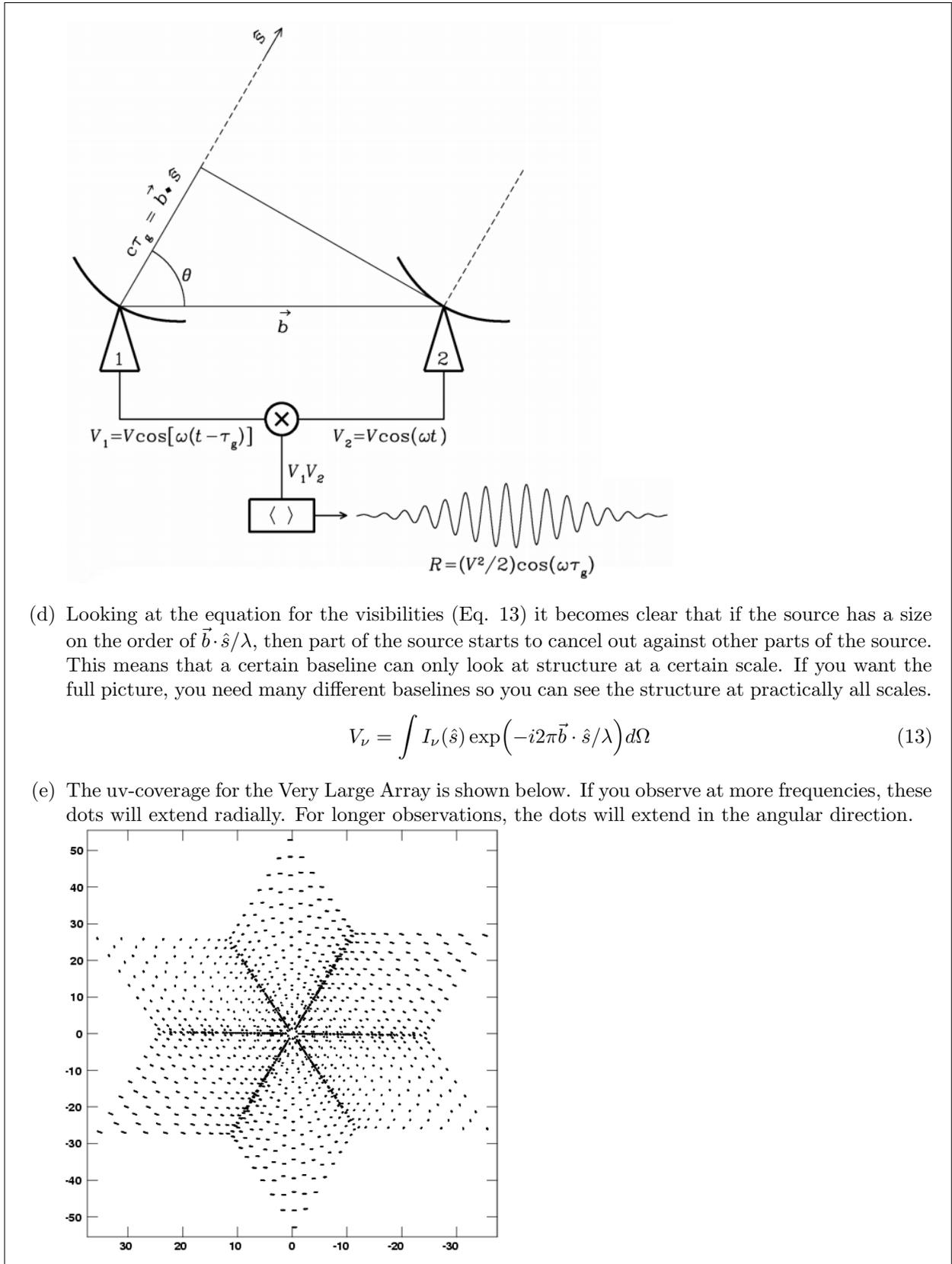
3. Question 3

Interferometry

- Describe what an interferometer is and state 3 advantages they have over single-dish antennae.
- The Very Large Array has 27 dishes. Calculate how much the sensitivity will improve by using all 27 dishes relative to a single dish.
- Draw a basic diagram of a two-element interferometer including correlator, and indicate where the geometric delay comes from.
- A silly 2nd-year student says it doesn't make any sense for an interferometer to have 27 dishes since your resolution is only determined by the longest baseline. Explain why it is necessary to have many dishes.
- Draw what the uv-coverage looks like for the Very Large Array for a single moment in time, and indicate what happens to the uv-coverage if you keep measuring for an hour and at multiple frequencies.
- You observe a perfect point source with an interferometer. What do you expect to see for the measured amplitudes as a function of baseline length?
- Explain what a dirty beam is and what it is used for.

Solution:

- An interferometer is a set of antennae with coupled outputs to create a single observation. They have better angular resolution than single-dish antennae. They also have better sensitivity, and they can have a larger field-of-view (in certain configurations).
- The sensitivity will improve by a factor $\sqrt{N(N-1)} \approx 26.5$.
- Here the two antennae (1 & 2) are connected by a correlator which digitally multiplies the signal. Then this multiplied signal is averaged over time to result in the response of the two antennae. The geometric delay comes from the fact that the radio waves have to travel different lengths to reach both antennae.





- (f) Since you observe a perfect point source, your interferometer will never cancel out part of the flux against another part. This means that your amplitudes will be independent of distance.
- (g) The dirty beam of an interferometer is analogous to a point-spread function. The dirty beam is the Fourier transform of the uv-coverage, and indicates the sensitivity of an interferometer as a function of direction. The direction dependence of the interferometer causes artifacts (distortions) in the image, which can be removed by deconvolving the image with the dirty beam. (A convolution is a multiplication in Fourier space.)

4. Question 4

Emission mechanisms

(a) Show that the total power emitted by an accelerated charge q is given by the following equation:

$$P = \frac{2}{3} \frac{q^2 a^2}{c^3} \tag{14}$$

(b) Assuming you have a electron energy distribution of

$$N(E)dE = DE^{-\delta}dE \tag{15}$$

and given that you have an emission coefficient of

$$\epsilon_\nu d\nu = -\frac{dE}{dt} N(E)dE \tag{16}$$

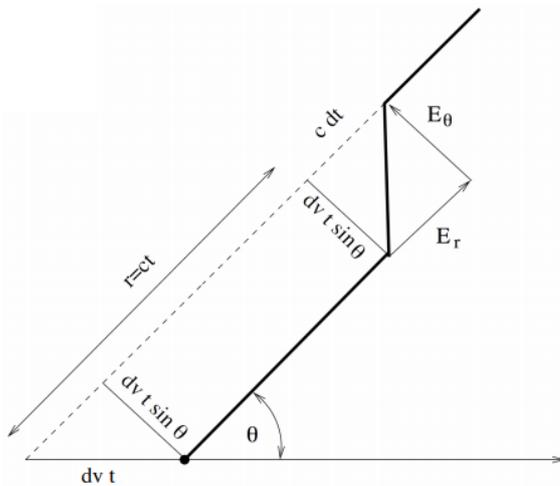
show that the resulting spectrum is of the form

$$S_\nu \propto \nu^\alpha. \tag{17}$$

You may use the approximation that electrons radiate away most of their energy at a frequency given by $\nu = \gamma^2 \nu_G$, and you may use the power emitted by a single relativistic electron: $P = \frac{4}{3} \sigma_T \beta^2 \gamma^2 c U_B$.

Solution:

(a) We start with a sketch of the electric field lines of a charge that accelerated for a short while.



From this graph, it is easy to find

$$\frac{E_\theta}{E_r} = \frac{dv t \sin \theta}{c dt}. \tag{18}$$



Given that $E_r = q/r^2$, we can write

$$E_\theta = \frac{q}{r^2} \left(\frac{dv}{dt} \right) \frac{r \sin \theta}{c^2} = \frac{q a \sin \theta}{r c^2}. \quad (19)$$

To get the power in a given direction, we need the Poynting flux:

$$\vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{H}. \quad (20)$$

In correct units (cgs), $|\vec{E}| = |\vec{H}|$, so given that \vec{E} and \vec{H} are always perpendicular we can easily write

$$|\vec{S}| = \frac{c}{4\pi} \left(\frac{q a \sin \theta}{r c^2} \right)^2 = \frac{q^2 a^2}{4\pi c^3 r^2} \sin^2 \theta. \quad (21)$$

If you integrate this over a sphere to get the total power, you get an extra factor r^2 , which cancels with the $1/r^2$. Also, you get a factor 2π for integration over the ϕ angle. Finally, you get an extra factor $\sin \theta$, and the integral over $\sin^3 \theta$ gives a factor $4/3$. In total, this gives

$$P = \frac{2}{3} \frac{q^2 a^2}{c^3} \quad (22)$$

(b) Given that electrons emit most of their energy at a frequency

$$\nu = \gamma^2 \nu_G \quad (23)$$

we can then write the energy of electrons as

$$E = \gamma m_e c^2 = \sqrt{\frac{\nu}{\nu_G}} m_e c^2 \quad (24)$$

Taking the derivative with respect to ν :

$$\frac{dE}{d\nu} = \frac{m_e c^2 \nu^{-1/2}}{2\nu_G^{1/2}} \quad (25)$$

This gives for the emission coefficient:

$$\epsilon_\nu = \left(\frac{4}{3} \sigma_T \beta^2 \gamma^2 c U_B \right) (K E^{-\delta}) \frac{m_e c^2 \nu^{-1/2}}{2\nu_G^{1/2}}. \quad (26)$$

Taking only the proportionality of this with ν , we get (from the γ^2 , the $E^{-\delta}$ and the $\nu^{-1/2}$):

$$\epsilon_\nu \propto \nu^{(1-\delta)/2}. \quad (27)$$

Finally, since the emission coefficient is proportional to the intensity, we find

$$S_\nu \propto \nu^{(1-\delta)/2} = \nu^\alpha. \quad (28)$$

5. Question 5

Fourier fun times! :D

(a) Perform a Fourier transform of a Dirac delta function: $f(x) = A\delta_D(x - d)$.

(b) How can this result be interpreted in terms of observing a point source with a 2-element interferometer?



- (c) What happens when you change the location/amplitude of the point source with respect to the telescope? Interpret your result.
- (d) Perform a Fourier transform of a top hat function of width D and height 1.
- (e) How can this Fourier transform be generalized for a rectangular source?
- (f) Using a drawing of the sinc function explain why it can be difficult to observe faint sources next to bright sources.

Solution:

(a)

$$\mathcal{F}(k) = \int_{-\infty}^{\infty} A\delta_D(x-d)e^{-2\pi ikx} dx = Ae^{-2\pi ikd} \quad (29)$$

(b) In that case we have to look at the amplitude of the solutions which is:

$$V = |Ae^{-2\pi ikd}| = A\cos(2\pi kd) \quad (30)$$

so the observed pattern for a two element interferometer of a point source is a wave.

- (c) The amplitude of the source (total observed intensity) can always be found at the top of one of the harmonic waves. The phase of the wave changes when you displace the source
- (d) The Fourier transform of a top hat of width 1 is given by

$$\begin{aligned} \mathcal{F}(k) &= \int_{-\infty}^{\infty} f(x)e^{-2\pi ikx} dx \\ &= \int_{-1/2}^{1/2} 1 \times e^{-2\pi ikx} dx \\ &= \frac{1}{-2\pi ik} (e^{\pi ik} - e^{-\pi ik}) \\ &= \frac{\sin(\pi k)}{\pi k} \\ &= \text{sinc}(k). \end{aligned}$$

Then we can use the Similarity Theorem, stating that if $\mathcal{F}(k)$ is the Fourier transform of $f(x)$, then the Fourier transform of $f(ax)$ is given by

$$\frac{1}{|a|} \mathcal{F}\left(\frac{1}{a}\right).$$

Therefore the Fourier transform of the top hat of width a is given by

$$\mathcal{F}(k) \propto \int_{-D/2}^{D/2} e^{-2\pi ikx} dx \propto \frac{\sin D\pi k}{D\pi k} \propto \text{sinc}(Dk)$$

(e) Working out the Fourier transform for two dimensions gives

$$\mathcal{F}(k) \propto \text{sinc}(Dk)\text{sinc}(Hk),$$

where D and H are the dimensions of the rectangle.

(f) If the source happens to be in one of the side lobes the strong signal will overrule the faint signal completely.