Mechanics and Relativity: M1

November 28, 2024, Aletta Jacobshal
Duration: 60 mins

Before you start, read the following:

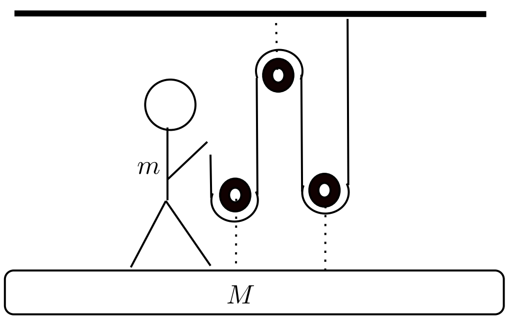
- There are 2 problems with subquestions, and you can earn 90 points in total (45 per problem). Your final grade is 1+(points)/10.
- Write your name and student number on all sheets.
- Make clear arguments and derivations and use correct notation. *Derive* means to start from first principles, and show all intermediate (mathematical) steps you used to get to your answer!
- Support your arguments by clear drawings where appropriate.
- Write your answers in the boxes provided. If you need more space, use the lined drafting paper.
- Generally use drafting paper for scratch work. Don't hand this in unless you ran out of space in the answer boxes.
- Write in a readable manner, illegible handwriting will not be graded.

Possibly relevant equations and values:

$$F = ma\,, \quad E = mc^2\,, K = \tfrac{1}{2}mv^2\,, \quad V = mgh\,, \quad V = -\tfrac{1}{2}kx^2\,, \quad g \approx 10m/s^2\,. \tag{1}$$

Question 1: A platform and three pulleys

A platform with a person on it is hanging from a ceiling, with a long rope (straight line) and three shorter ropes (dotted lines) connected to three pulleys, as illustrated in the picture. The person standing on the platform is pulling the long rope. The platform and person have masses M and m, respectively, and you can consider the pulleys to be massless and frictionless. You only have to consider vertical motion in this question and you don't have to worry about rotation.



(a) **(10 pts)** Draw the free-body diagrams separately for the different objects of this configurations (that is, the person, platform and the three pulleys), clearly indicating which forces act on which objects.

(b) **(15 pts)** What force does the person need to exert on the rope to achieve a static configuration? Express your answer in terms of M, m and g.

(c) **(10 pt)** What is the increase in potential energy of the entire configuration when the person would pull in a length L of the rope?

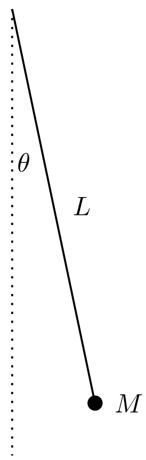
(d) **(10 pt)** Finally, suppose we would include a small amount of friction in the three pulleys. Would this increase, decrease or not affect the (minimal) force that the person needs to exert to achieve a static configuration? Also indicate the implications of friction on the tension throughout the rope: is it constant throughout the rope or not? If not, which part of the rope has the highest tension? (No explanation needed.)

Question 2: Simple pendulum

Consider a simple pendulum consisting of a mass M hanging from a rope of length L under an angle θ. For small oscillations, Newton's second law for the angle reads

$$\ddot{\theta} = -\omega^2\theta - 2\gamma\dot{\theta}\,, \tag{2}$$

where we have included a damping force (that can correspond to the pendulum moving through a viscous fluid) with coefficient γ and where $\omega^2 = g/L$.



(a) **(15 pts)** The general solution for small oscillations will involve an overall exponentially decaying factor, and hence is of the form $\theta(t) = e^{-\gamma t} f(t)$. Derive the second order differential equation that governs the function $f(t)$; i.e. which equation of the form $\ddot{f} + \ldots = 0$ does the time-dependent function $f(t)$ satisfy?

(b) **(15 pts)** What is the most general solution to the underdamped case for the simple pendulum above? You can give your answer in terms of either the angle $\theta(t)$ or the function $f(t)$; no derivation needed.

(c) **(15 pts)** Sketch the angle $\theta(t)$ as a function of time for the three different cases corresponding to the undamped, underdamped and overdamped harmonic oscillators. Clearly indicate qualitative differences in e.g amplitudes and/or time scales. For all three cases, take identical initial conditions where the pendulum is moving through the vertical with positive $\dot{\theta}$ at $t = 0$.