Name: Student Number:

Mechanics and Relativity: M1

December 1, 2023, Aletta Jacobshal
Duration: 60 mins

Before you start, read the following:

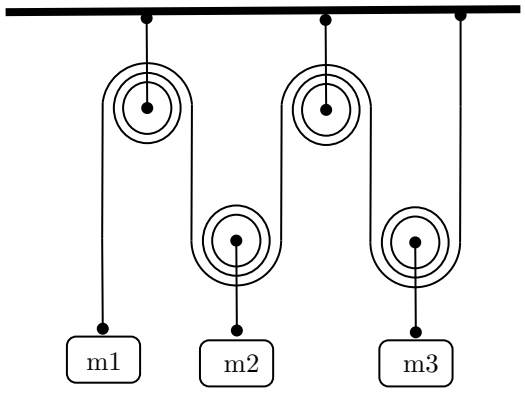
- There are 2 problems with subquestions, and you can earn 90 points in total (45 per problem). Your final grade is 1+(points)/10.
- Write your name and student number on all sheets.
- Make clear arguments and derivations and use correct notation. *Derive* means to start from first principles, and show all intermediate (mathematical) steps you used to get to your answer!
- Support your arguments by clear drawings where appropriate.
- Write your answers in the boxes provided. If you need more space, use the lined drafting paper.
- Generally use drafting paper for scratch work. Don't hand this in unless you ran out of space in the answer boxes.
- Write in a readable manner, illegible handwriting will not be graded.

Possibly relevant equations and values:

$$F = ma\,, \quad E = mc^2\,, K = \tfrac{1}{2}mv^2\,, \quad V = mgh\,, \quad V = -\tfrac{1}{2}kx^2\,, \quad g \approx 10m/s^2\,. \tag{1}$$

Question 1: A double Atwood machine

Consider an extended Atwood machine as indicated in the picture, with a single rope connecting three masses m_i. You can assume the rope is massless and does not stretch, and the pulleys are massless and have no friction.



(a) **(15 pts)** Which ratios of the three masses are needed to achieve a static configuration?

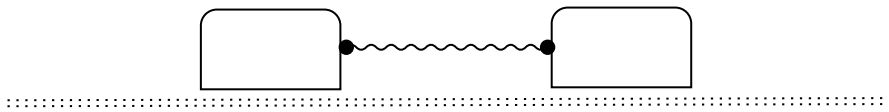
(b) **(15 pts)** Which acceleration does m_1 have when all three masses are equal?

(c) **(15 pt)** Is the combination of potential energy and kinetic energy of the first mass m_1 conserved when it accelerates as in (b) (so with all masses equal)? Show how this follows from a calculation. (If you did not find an answer at (b), take the acceleration to be half of the gravitational one, i.e. $a = g/2$.) Explain in one or two sentences whether this combination of energy should be conserved.

Name: Student Number:

Question 2: Normal modes of two carts

Consider rails (of infinite extent) with two carts (of equal masses m) that are connected by a spring with spring constant k, see the picture.



(a) **(15 pts)** What are Newton's 2nd laws for the two carts, expressed in terms of their locations $x_{1,2}$? Give the explicit form of these differential equations for the first cart and for the second cart.

(b) **(15 pts)** Derive what are the normal modes (as linear combinations of the locations $x_{1,2}$ of the two carts) of this system and the differential equations governing their dynamics. Moreover, what is the most general solution of this system? *Hint: when expressed in terms of normal modes, Newton's 2nd laws become two separate ODE's for the two combinations.*

(c) **(15 pts)** Now include a friction term due to e.g. a viscous fluid in which the system is immersed. It is modelled by a drag force $-2m\gamma\dot{x}_{1,2}$ (with γ constant) that is proportional to the velocities of the carts. How does this change the differential equations governing the normal modes? What is the solution in this case (with a small friction term, i.e. the underdamped case)?