Name: Student Number:

Mechanics and Relativity: M2

December 19, 2023, Aletta Jacobshal
Duration: 90 mins

Before you start, read the following:

- There are 3 problems with subquestions, and you can earn 90 points in total. Your final grade is 1+(points)/10.
- Write your name and student number on all sheets.
- Make clear arguments and derivations and use correct notation. *Derive* means to start from first principles, and show all intermediate (mathematical) steps you used to get to your answer!
- Support your arguments by clear drawings where appropriate.
- Write your answers in the boxes provided. If you need more space, use the lined drafting paper.
- Generally use drafting paper for scratch work. Don't hand this in unless you ran out of space in the answer boxes.
- Write in a readable manner, illegible handwriting will not be graded.

Possibly relevant equations and values:

$$F = ma\,, \quad \vec{L} = \vec{r} \times \vec{p}, \quad T = \tfrac{1}{2} I_z \omega^2\,, \quad \tau = I\alpha\,. \tag{1}$$

Question 1: Central forces

(a) **(10 pts)** What is the requirement for a force to be central? You can phrase this requirement either in terms of the force $\vec{F}$ or its potential energy V (either is fine).

(b) **(10 pts)** Prove that a central force preserves angular momentum.

(c) **(10 pts)** A prominent example of a central force is Newton's law of gravity that governs the attraction between the Sun and other celestial bodies. The resulting trajectories include ellipses and hyperbola. What is the difference between these types of trajectories in terms of their total energy; in other words, how can one distinguish these based on their energy?

Question 2: Moment of inertia

(a) **(15 pts)** Calculate the moment of inertia of a massive ring of total mass M and radius R, where all mass is located at the rim. Consider the case where the ring lies in the horizontal plane and the axis of rotation is vertical and goes through the center of mass.

(b) **(10 pts)** Now consider the same set-up, but now the axis of rotation goes through the outer edge of the ring (and it still vertical). What is the difference between the moments of inertia of this set-up and that of question (a)? (You can answer this question also when you didn't get an answer at the previous question.)

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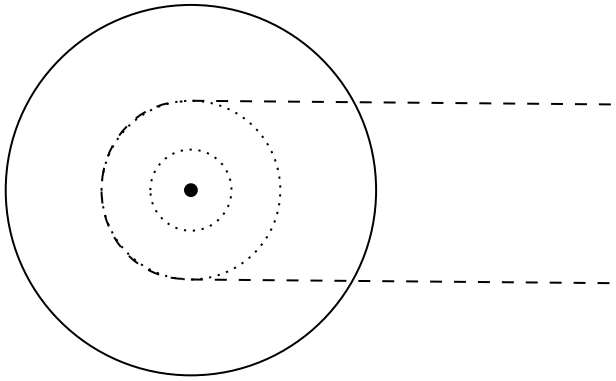
Question 3: Bikes and gears

Consider a bike with different gears on the rear wheel. For simplicity we will take the rear wheel to have a radius R, and the bike chain can go via either a gear wheel of radius $R/2$ or one of $R/4$. In the picture, the use of the gear wheel of radius $R/2$ is illustrated.



(a) **(10 pts)** Suppose a person is exerting a fixed force on the chain (via the pedals). Which of the two gear wheels will lead to the largest acceleration of the bike, the one of radius $R/2$ or $R/4$, or are they the same? If different, indicate the ratio of the accelerations in the two cases. Briefly explain your answer using the notion of torque.

(b) **(10 pts)** Suppose the acceleration remains constant as the person speeds up from 0 to 30 km/h. How does the work exerted by the person compare in the two cases, using the different gear wheels - which one is larger, or are they the same? Briefly explain your answer.

(c) **(15 pts)** Express the total energy of the moving bike in terms of its velocity v, its mass M and the wheel radius R. Use the approximation that half of the mass is in the frame of the bike plus the driver, and the other half is in the masses of the wheels (all at the rim), i.e. either wheel carries 25 percent of the mass. You can use your answer under question (2a) here (if you didn't get an answer there, you can use $I = \frac{4}{9}MR^2$ as an answer for question (2a)).