Name: Student Number:

Mechanics and Relativity: M3

January 22, 2024, Aletta Jacobshal
Duration: 120 mins

Before you start, read the following:

- There are 3 problems with subquestions, and you can earn 90 points in total. Your final grade is 1+(points)/10.
- Write your name and student number on all sheets.
- Make clear arguments and derivations and use correct notation. *Derive* means to start from first principles, and show all intermediate (mathematical) steps you used to get to your answer!
- Support your arguments by clear drawings where appropriate.
- Write your answers in the boxes provided. If you need more space, use the lined drafting paper.
- Generally use drafting paper for scratch work. Don't hand this in unless you ran out of space in the answer boxes.
- Write in a readable manner, illegible handwriting will not be graded.

Possibly relevant equations:

$$\vec{F} = m\vec{a}\,, \quad \vec{L} = \vec{r} \times \vec{p}\,, \quad \vec{\tau} = \vec{r} \times \vec{F}\,, \quad \vec{F}_{\text{centr}} = -m\vec{\omega} \times (\vec{\omega} \times \vec{r})\,, \quad \vec{F}_{\text{Cor}} = -2m\vec{\omega} \times \vec{v}\,,$$

and the trigonometric identities $\sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta))$ and $\cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta))$.

Question 1: Principal axes

(a) **(10 pts)** Consider a ring standing up vertically, with mass M and radius R, which is so flat that it's effectively a planar object. Indicate the three principal axes for rotation around the lowest point of the ring in one or several clear drawings.

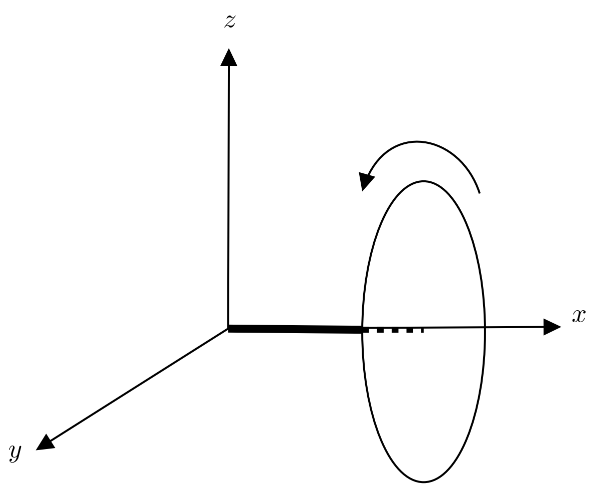
(b) **(10 pts)** Of the three principal moments, associated with the object and principal axes of the previous subquestion, calculate the one that has the *smallest* value.

(c) **(10 pts)** Now instead consider rotation around the center of mass of the ring. Will all three principal moments have different values, or will some be equal? Briefly explain your answer, e.g. in one or two sentences. (No calculations needed.)

Question 2: Suspended bike wheel with spin and precession

Consider a bike wheel that is suspended in mid air on one of its sides, allowing it to spin and precess. Take all mass M to be at the rim of the wheel at radius R, such that its moment of inertia around the axis of the wheel is MR^2. Take the distance from the center of mass to the suspension point to be x_0. Finally, take the spinning direction to be clockwise as seen from the suspension point with angular frequency ω_3. You can neglect friction.

(a) **(15 pts)** Using the torque and rate of change of angular momentum, derive the angular precession frequency Ω of the bike wheel, and its direction.



(b) **(15 pts)** The total angular momentum $\vec{L}$ of any body is always composed of the angular momentum *of* the center of mass, $\vec{R} \times \vec{P}$, and the angular momentum *around* the center of mass, $\vec{L}_{\text{CM}}$. In the above situation, what are the magnitudes of both components of angular momentum? Also, clearly indicate their directions in a drawing.

Question 3: Fictitious forces

Imagine you live somewhere on Earth at a latitude of 45 degrees, so exactly in between the North Pole and the equator. Take the Earth to be a perfect sphere with a radius of 6400 km and an angular speed of $\omega = 7 \cdot 10^{-5}$ 1/s. Consider a cloud above your house moving westwards with speed v, and restrict its motion to the surface of the Earth (i.e. no vertical motion).

(a) **(10 pts)** Out of the three fictitious forces given by translation, centrifugal and Coriolis,

$$\vec{F}_{\text{fict}} = -m\ddot{\vec{R}}_S - m\vec{\omega} \times (\vec{\omega} \times \vec{r}) - 2m\vec{\omega} \times \vec{v}\,, \tag{1}$$

which one(s) will act on this (otherwise free) cloud? Calculate their magnitude(s) (in terms of v and ω) and their direction(s), in the plane spanned by the surface of the Earth.

(b) **(10 pts)** For this cloud to trace out a circle around the Earth at constant latitude, it needs to be subject to a net force pointing north with magnitude $\frac{1}{2}m\omega^2 R$ (acting as centripetal force). Can the fictitious forces give rise to such a net force, what condition(s) need to be met for this (for instance in terms of the speed v)?

(c) **(10 pts)** Now consider the Earth from the perspective of an inertial observer located in outer space. Imagine that the center of mass of the Earth is at a constant location in this frame (and thus neglect the effect of the Moon, Sun etc.). What kind of motion does the cloud display as measured by this observer?