Name: Student Number:

Mechanics and Relativity: M3

January 24, 2023, Aletta Jacobshal
Duration: 120 mins

Before you start, read the following:

- There are 3 problems with subquestions, and you can earn 90 points in total. Your final grade is 1+(points)/10.
- Write your name and student number on all sheets.
- Make clear arguments and derivations and use correct notation. *Derive* means to start from first principles, and show all intermediate (mathematical) steps you used to get to your answer!
- Support your arguments by clear drawings where appropriate.
- Write your answers in the boxes provided. If you need more space, use the lined drafting paper.
- Generally use drafting paper for scratch work. Don't hand this in unless you ran out of space in the answer boxes.
- Write in a readable manner, illegible handwriting will not be graded.

Possibly relevant equations:

$$\vec{F} = m\vec{a}\,, \quad \vec{L} = \vec{r} \times \vec{p}, \quad \vec{\tau} = \vec{r} \times \vec{F}\,, \quad \vec{F}_{\text{centr}} = -m\vec{\omega} \times (\vec{\omega} \times \vec{r})\,, \quad \vec{F}_{\text{Cor}} = -2m\vec{\omega} \times \vec{v}\,, \quad \vec{F}_{\text{azim}} = -m(\frac{d\vec{\omega}}{dt}) \times \vec{r}\,,$$

and the Taylor expansion $(1 + ax)^b \approx 1 + abx + \ldots$ at small x.

Question 1: Principal axes

(a) **(10 pts)** According to Chasles' theorem, how does one characterize the most general behaviour (at a given moment) of a solid object with reference to an arbitrary point P inside that body? Which vector quantities does this require?

(b) **(10 pts)** Consider an object that has the shape and orientation of a door, and assume its density to be constant. One of the principal axes with respect to the door's center of mass is the vertical direction. Is this also a principal axis with respect to other points of the door that lie along this vertical direction? Briefly explain your answer.

(c) **(10 pts)** Now instead imagine a random shape in 3D, e.g. a potato-like object with no symmetries. Pick a random plane that goes through the center of mass of the object. How many principal axes of the object with respect to the center of mass would you expect to lie in this plane? Briefly explain your answer.

Question 2: Spinning top without fixed pivot point on table

Consider a spinning top that is spinning and precessing at a table, without its bottom point being fixed to the table - instead, the bottom point can move freely and without friction. Take the top to be spinning clockwise (as seen from above). It has a center of mass at a distance ℓ from its bottom point, and makes an angle θ with respect to the vertical direction. Its moment of inertia around the spinning direction is I_3, and it is spinning with angular frequency ω_3. Throughout this exercise, you can use the fast-spinning approximation (in which angular momentum is dominated by spinning) and you only have to consider vertical forces.

(a) **(15 pts)** With respect to the center of mass of the spinning top, which force(s) induce(s) a torque? What is the magnitude and direction of this torque? Moreover, what is the magnitude and direction of angular momentum? Indicate in a planar drawing in which direction the horizontal component of angular momentum and its rate of change are pointing.

(b) **(15 pts)** The bottom point of the top will trace out a circle on the table. Derive the radius, frequency and orientation of this motion.

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Question 3: Dropping a ball

Imagine you live somewhere on earth at a latitude of 45 degrees, so exactly in between the North Pole and the equator. Approximate the Earth to have a radius of 6400 km, an angular speed of $\omega = 7 \cdot 10^{-5}$ 1/s and a gravitational acceleration of $g = 10$ m/s^2. We will be dropping a ball from a height of five meters, resulting in a drop time of one second.

(a) **(15 pts)** We will first consider the West-East deflection. Which fictitious force is responsible for this? Calculate the deflection (one significant digit) and indicate its direction.

(b) **(15 pts)** Now we turn to the North-South deflection. Which fictitious force is responsible for this? Calculate the deflection (one significant digit) and indicate its direction.