Conceptual Questions Week 8

Oktober 27 2022

What is the difference between locality and causality?

- Locality: Action at a distance / instantaneous effect
- Causes between effects have to be in the lightcone
  - No infomation faster transferred faster than the speed of light

Is special relativity (as an extension of classical mechanics, ie without quantum effects) local and causal?

- Special relativity: Local and Causal
- Quantum Mechanics: non-local and causal. (no information transferred with collapsing spin up spin down experiment)

The ground state of helium has 2 electrons. The ground state wavefunction of the helium electrons can be written as a product of the hydrogen wavefunctions that we derived in section 4.2, in other words with the exponential fall-off.
Discuss the symmetry of the spatial wavefunction, the spin part and the total wavefunction. Is the ground state of this system in the singlet or triplet configuration?

- Ground state wavefunction symmetric:

$$\psi_0(\mathbf{r}_1, \mathbf{r_2}) = \psi_{lmn}(\mathbf{r}_1)\psi_{l'm'n'}(\mathbf{r}_2) = \psi_{100}(\mathbf{r}_1)\psi_{100}(\mathbf{r}_2) = \frac{8}{\pi a^3} e^{-2(r_1+r_2)/a}$$

$$\begin{cases} |1\ 1\rangle &= \uparrow\uparrow \\ |1\ 0\rangle &= \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \quad \textit{triplet} \\ |1\ -1\rangle &= \downarrow\downarrow \end{cases}$$

$$\boxed{\left\{|0\ 0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \quad \textit{singlet}\right.}$$

Does the energy of the ground state increase or decrease when we take Coulomb electron repulsion into account?

- The energy increases. The contribution to the Hamiltonian is positive:

$$H = \left\{ -\frac{\hbar^2}{2m}\nabla_1^2 - \frac{1}{4\pi\epsilon_0}\frac{2e^2}{r_1} \right\} + \left\{ -\frac{\hbar^2}{2m}\nabla_2^2 - \frac{1}{4\pi\epsilon_0}\frac{2e^2}{r_2} \right\} + \frac{1}{4\pi\epsilon_0}\frac{e^2}{|\overrightarrow{r_1} - \overrightarrow{r_2}|}$$

What would the energy level scheme of helium look like (qualitatively) if the electrons were identical bosons? Pretend these "electrons" still have spin 1/2, so the spin configurations are the singlet and triplet.

Ground state:

- The symmetrization requirement for bosons says that the total boson spatial wavefunction always has to be symmetric.
- Spatial wavefunction is still symmetric
- Spin wavefunction now also has to be symmetric → triplet state → orthohelium → triple degenerate

Excited states:

$$\psi(r) = \psi_{nlm}\psi_{100}$$

- ortho (triplet) (when spatial wavefunction is symmetric)
  - Symmetric spatial wave function
  - Higher energy than antisymmetric spatial wave function.
    - Because electrons closer together, higher interaction energy
- Para (singlet) (when spatial wavefunction is antisymmetric)