Quantum Physics 1 - Homework 1

Due on saturday morning (Sept 10) at 12:00.

1. [0.2pt.$\times 10 = 2$pt.] For each of the following wave functions, write down whether it can correspond to a physically realisable state for a particle if $A \in \mathbb{R}_{++}$ is some constant? Write down a one-line explanation for each wave function to support your answer.

a. $\psi(x) = A$;

b. $\psi(x) = Ae^{-x}$;

c. $\psi(x) = Ae^{-|x|}$;

d. $\psi(x) = Ae^{-x^2}$;

e. $\psi(x) = A/x$ on $x \in [1, \infty)$;

f. $\psi(x) = A/\sqrt{x}$ on $x \in [1, \infty)$;

g. $\psi(x) = Ae^{-[\log(x)]^2}$ on $x \in (0, \infty)$;

h. $\psi(x) = A\sin(x)$ on $x \in [-\pi, \pi]$;

i. $\psi(x) = A\sin(x)/x$;

j. $\psi(x) = A[\cos(x) + i\sin(x)]$.

2. [8pt.] Consider a wave function of a particle of mass m at $t = 0$ given by

$$\Psi(x, 0) = \begin{cases} A(e^{ikx} + e^{-ikx}) & \text{if } -\frac{\pi}{2k} \leq x \leq \frac{\pi}{2k} \\ 0 & \text{otherwise} \end{cases}.$$

a. [1pt.] Find the normalisation constant A and sketch the wave function.

b. [1pt.] What is the probability that the particle can be found on the interval $0 \leq x \leq \frac{\pi}{2k}$?

c. [2pt.] Calculate the standard deviation of position x; σ_x.

d. [2pt.] Calculate the standard deviation of momentum p; σ_p.

e. [1pt.] Does this particle obey Heisenberg's famous position-momentum uncertainty principle?

f. [1pt.] Find the expectation value of the kinetic energy of the particle $\langle T \rangle$. How is the spatial width of the wavefunction related to $\langle T \rangle$? How can you explain the dependence, which you find?

Grade = your points.