# Quantum Physics 1 - Homework 2

Due on Saturday morning (Sept 17) at 12:00.

**1. Quantum Harmonic Oscillator** [10pt.]

A particle of mass $m$ is trapped in the harmonic potential $V(x) = \frac{1}{2}m\omega^2 x^2$, with $\omega$ being the frequency of oscillation. The normalised wavefunction of this particle at the time $t = 0$ is given by:

$$\Psi(x,0) = \frac{1}{2}\sqrt[4]{\frac{m\omega}{\pi\hbar}}\ [b_+ + ib_-]\ e^{-\frac{m\omega}{2\hbar}x^2}\ ,$$

with $b_\pm = 1 \pm x\sqrt{\frac{2m\omega}{\hbar}}$. Note that $b_\pm$ is a function of $x$.

a. [2pt.] The given wavefunction $\Psi(x,0)$ can be expressed in terms of the wavefunctions $\psi_n$ of the quantum harmonic oscillator as follows:

$$\Psi(x,0) = \sum_{n=0}^{\infty} c_n\ \psi_n(x). \tag{1}$$

The coefficients $c_n$ can be determined using Fourier's trick:

$$c_n = \frac{1}{\sqrt{n!}}\int \psi_0\ (\hat{a}_-)^n\ \Psi(x,0)\ dx. \tag{2}$$

Calculate $c_0$ and $c_1$ explicitly using Fourier's trick.

*Hint: Before trying to evaluate any integrals, check whether they're even or odd.*

b. [1pt.] Calculate $\sum_{n=0}^{1}|c_n|^2$. Does your answer make sense? Explain why. (If you didn't manage to solve question (a), you can take $\Psi(x,0) = \frac{1}{\sqrt{2}}(\psi_0 + i\psi_1)$ for the subsequent calculations.)

c. [1pt.] What happens when you try to calculate $c_2$ or higher coefficients?

d. [1pt.] Construct $\Psi(x,t)$ by adding the time-dependence to $\Psi(x,0)$.

e. [2pt.] Calculate $\langle x\rangle$ for the time-dependent wavefunction.

f. [2pt.] We have already seen that we can calculate the expectation value of momentum, $\langle p\rangle$, to get an idea of how the expectation value of position, $\langle x\rangle$, moves over time. Perhaps we're interested in calculating $\langle a\rangle$ today, the expectation value of acceleration:

$$\langle a\rangle = \frac{1}{m}\frac{i}{\hbar}\langle\ [V,\ \hat{p}]\ \rangle. \tag{3}$$

First, calculate the commutator $[V,\ \hat{p}]$ and substitute $V = \frac{1}{2}m\omega^2x^2$.

*Hint: Follow the same steps as in the book by introducing the 'test function' f.*

g. [1pt.] Calculate $\langle a\rangle$ for this wavefunction. How does your result relate to classical mechanics?

*Grade = your points rounded to the nearest integer in {1, 4, 7, 10}.*