Quantum Physics 1 - Homework 3

Double Delta-Function Potential [10.5pts.]

We are going to take a look at some of the possible behaviours of a particle of mass m in a one-dimensional double-delta potential given by:

$$V(x) = -\alpha\delta(x) - \alpha\delta(x - x_0), \tag{1}$$

where α and x_0 are positive real numbers.

1. (0.5pts.) Sketch what this potential looks like.
2. (1.5pts.) How many distinct bound states are permitted in potential given by equation 1? Sketch all options and label them $\psi_1, \psi_2, \ldots$. (You don't have to find the mathematical description of their wavefunctions.)
3. (1pt.) In which points or intervals is there a non-zero probability of finding a particle in a bound state? (You can discuss each bound state individually if necessary.) How does this compare to a classical particle?
4. (0.5pts.) What range of energies do the scattering states correspond to?

In the next few steps, we will derive the reflection and transmission coefficients for some of the states. Consider scattering states that have a wavenumber (and hence momentum) given by $k = n\pi/x_0$ with n an arbitrary positive integer.

5. (1.5pts.) Write down the most general spatial wavefunction corresponding to an incoming scattering state **from the left** (i.e. from negative infinity). Is this wavefunction normalisable? Do not worry about time-dependence, boundary conditions and calculating normalisation for the moment.
6. (1.5pts.) What boundary conditions need to be imposed?
7. (2pts.) Now that you have the potential, wavefunction and boundary conditions, derive the transmission and reflection coefficients as a function of α.
8. (1pt.) In general, do the scattering states of this potential have discrete energy levels? In the questions above we set $k = n\pi/x_0$ with $n \in \mathbb{N}$. Is this the correct/only possible quantisation of the energies of the scattering states?

An arbitrary wavefunction can be written as a linear combination of functions from a complete set of eigenstates.

9. (1pt.) Write down the general formula for an arbitrary wavefunction $\Psi(x, t)$ using the stationary states of the potential given by equation 1. Don't forget to include time dependence. You don't have to find the exact energies and you can assume that the stationary states of this potential form a complete set. *Hint: Check out equation 2.100 (2nd Ed) or 2.101 (3rd Ed)*

Grade = 10 × percentage of your points rounded to the nearest integer in {1, 4, 7, 10}.