Quantum Physics 1 - Homework 5

Due on Monday (Oct 11) at 11:59 AM

1. [2pt.] Are the following statements true or false? (no explanation required)

(a) A state cannot be a simultaneous eigenstate of two compatible observables.

(b) Hermitian operators have real eigenvalues in finite-dimensional vector spaces.

(c) All wavefunctions are linear combinations of eigenfunctions of a Hermitian operator.

(d) The generalised uncertainty principle is one of the central assumptions of quantum mechanics.

(e) If a matrix is equal to the complex conjugate of its transpose, then the matrix is Hermitian.

(f) All operators which correspond to physical observables are Hermitian.

2. [3pt] Recall that the groundstate wavefunction for the harmonic oscillator is given by:

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2} \tag{1}$$

Calculate the expectation value of the product of the position and momentum operator, $\langle xp \rangle$, in the ground state of the harmonic oscillator. What does the answer tell you about the hermiticity of the $\hat{x}\hat{p}$ operator?

3. [2+1+1= 4pt.] The Hamiltonian for a certain three-level system is

$$\hat{H} = \epsilon\Big(|\alpha\rangle \langle\alpha| + |\alpha\rangle \langle\gamma| + |\beta\rangle \langle\beta| + |\gamma\rangle \langle\alpha| + |\gamma\rangle \langle\gamma| \Big), \tag{2}$$

where $|\alpha\rangle$, $|\beta\rangle$, $|\gamma\rangle$ is an orthonormal basis (in Dirac's bra-ket notation) and ϵ is a nonzero number with the dimensions of energy.

(a) Taking $\hat{H}|\alpha\rangle = \epsilon(|\alpha\rangle + |\gamma\rangle)$ tells us that $|\alpha\rangle$ is **not** an eigenstate of the Hamiltonian. Find the normalised eigenstates and corresponding eigenvalues of the Hamiltonian. *(Hint: Write the general eigenvector as $|\psi\rangle = c_\alpha |\alpha\rangle + c_\beta |\beta\rangle + c_\gamma |\gamma\rangle$)*

(b) What is the matrix $\mathbf{H}$ representing $\hat{H}$ with respect to the $\{|\alpha\rangle, |\beta\rangle, |\gamma\rangle\}$ basis?

(c) Denote the time evolution of the eigenstates.