Quantum Physics 1 - Homework 7

Due on Monday (Oct 24th) at 11:59 AM

1a) [1pt.] Show that the operators $\hat{A}$ and $\hat{B}$ commute if the matrices A and B representing them have the same eigenvectors, and these eigenvectors span the vector space.

b) [0.5pt.] What does your result in a) tell you about the value of the commutator $\left[\hat{S}^2, \hat{S}_z\right]$?

c) [1pt.] What is the trace of the Pauli matrices? And how does it relate to their eigenvalues?

2a) [0.5pt.] What are the possible states $|sm\rangle$ for a particle with s = 3/2?

b) [1pt.] Use the results of exercise 4.53 (2nd edition) or 4.62 (3rd edition) to construct the matrices representing $\hat{S}_x$ and $\hat{S}_z$ for a particle with $s = 3/2$.

c) [1.5pt.] What are the eigenvalues of the matrix S_x you constructed in the previous question? If you didn't get an answer in b) you can use S_y instead:

$$S_y = \frac{i\hbar}{2}\begin{pmatrix} 0 & -\sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & -2 & 0 \\ 0 & 2 & 0 & -\sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix} \tag{1}$$

d) [0.5pt.] Explain qualitatively why the eigenvalues of S_x, S_y and S_z have to be the same.

e) [2pt.] Find the normalised eigenspinors of S_x, you can use S_y if you didn't find an answer in b).

f) [1.5pt.] The $\left|\frac{3}{2}\frac{3}{2}\right\rangle$ state can be decomposed into the eigenspinors of S_x (or S_y) using

$$\left|\frac{3}{2}\frac{3}{2}\right\rangle = c_1\chi_1^{(x)} + c_2\chi_2^{(x)} + c_3\chi_3^{(x)} + c_4\chi_4^{(x)}, \tag{2}$$

where $\chi_n^{(x)}$ are the eigenspinors of S_x for the eigenvalues λ_n. What are the constants c_n? Check if $\sum |c_n|^2 = 1$. If you didn't get an answer in e) you can use:

$$\chi_1 = \begin{pmatrix} 1 \\ \sqrt{2} \\ \sqrt{2} \\ 1 \end{pmatrix} \qquad \chi_2 = \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -1 \end{pmatrix} \qquad \chi_3 = \begin{pmatrix} 1 \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 1 \end{pmatrix} \qquad \chi_4 = \begin{pmatrix} 1 \\ -\sqrt{2} \\ \sqrt{2} \\ -1 \end{pmatrix} \tag{3}$$

$Grade \in \{1, 4, 7, 10\}$.