QUANTUM PHYSICS I - Nov. 3, 2022

Write your name and student number on **all** sheets. There are 3 problems in this two-hour exam, with 90 points in total. Questions indicated with a $(*)$ can involve lengthier calculations (and hence could be skipped in a first round). You are allowed to use the Griffiths book for consultation.

Problem 1: INFINITE CUBIC WELL (5+5+10+10 = 30 points)

Consider a quantum system with a 3D potential energy that is infinite outside of a cube of lengths a, and that is zero inside the cubic well.

a) Write down the normalised, time-dependent wavefunction for a particle in the groundstate of such a cubic well. Clearly indicate the dependence on t and the three spatial coordinates.

b) What are the degeneracies of the first and second excited states? Indicate the possible values the relevant quantum numbers can take.

c) Now consider that there are two particles in this cubic well, and moreover they are electrons with spin-1/2. For the moment we will take them to be non-interacting, i.e. we ignore the electron-electron Coulomb interaction. Write down the wavefunction for the groundstate of these two particles in this cubic well. Clearly indicate the dependence on the three spatial coordinates of both particles, as well as their spin state. You don't have to include time-dependence or worry about normalisation.

d) How does the energy of the ground state change when taking the electron-electron Coulomb interaction; is it higher, equal or lower as compared to the case where one neglects this interaction? Briefly explain your answer (max 3 sentences). Compare to the real-life system of the Helium atom.

Problem 2: HARMONIC OSCILLATOR (5+10+5+10 = 30 points)

Consider a single particle in the 1D harmonic oscillator system. Its wavefunction at $t = 0$ is taken to be

$$\psi = a\psi_0 + b\psi_2 , \quad |a|^2 + |b|^2 = 1 , \tag{1}$$

where ψ_0 is the normalised ground state, and ψ_2 is the normalised second excited state. a and b are complex coefficients subject to the above normalisation condition.

a) Write down the time-dependence of the above wavefunction. Is this a stationary state? Briefly explain (max 3 sentences) why (not).

b) Calculate the standard deviation at $t = 0$ for the operator x for a particle in this state, with the specific coefficient choice $a = \sqrt{1/3}$ and $b = \sqrt{2/3}$. $(*)$

c) Now consider the standard deviation at all times for the operator p for a particle in this state in the limit where a vanishes (you don't have to explicitly calculate this). Does this standard deviation vanish or not, and is it time-dependent or not? Briefly explain (max 3 sentences) your answers.

d) Calculate the commutator between the operators x^2 and p^2. Explain its relevance for the generalised Heisenberg uncertainty principle. Using this formula, briefly explain (max 3 sentences) whether we can measure x^2 and p^2 simultaneously with arbitrarily good accuracy.

Problem 3: EPR PARADOX (5+5+10+10 = 30 points)

Consider the decay of a pion into an electron and a positron, as proposed in the EPR set-up.

a) Briefly explain (max. 3 sentences) why this results in an entangled state.

b) Suppose the electron flies off to the left along the y-axis, while the positron goes to the right along the y-axis. Both spin detectors are then placed in the orthogonal (x, z)-plane. Suppose one of them is aligned with the positive z-axis, while the other is aligned with positive x axis. Write down all possible resulting measurements, with the corresponding probabilities.

c) Suppose the first detector along the positive z-axis measures spin-up for the electron. The second detector is rearranged within the (x, z)-plane and now is algined with the positive x-axis. Determine what the spin configuration of the positron is, and express it in terms of eigenstates of the spin operator corresponding to the current orientation of the second detector.

d) Suppose the second detector is again rearranged in the (x, z)-plane and now makes an angle θ with the positive z-axis. After measuring the electron spin to be up with respect to the z-axis, the spin state of the positron is given by

$$\psi_{e^+} = 0.6|\uparrow\rangle_\theta - 0.8|\downarrow\rangle_\theta\,, \tag{2}$$

in terms of eigenstates of the new orientation. What are the possible measurements and their probabilities in this case, and what does the product of both measurements average into? What can you conclude about the angle θ?