Quantum Physics 1

exam: Nov. 3, 2022.

1a] $\psi(x,y,z,t) = \underset{1pt}{\left(\frac{2}{a}\right)^{3/2}} \cdot \underbrace{\sin\left(\frac{\pi x}{a}\right) \cdot \sin\left(\frac{\pi y}{a}\right)}_{3pt} \cdot \sin\left(\frac{\pi z}{a}\right) \cdot e^{-iEt/\hbar}$ / 1pt.

$E = \frac{3}{2}\frac{\hbar^2 \pi^2}{ma^2}$

b) first excited: $(n_x, n_y, n_z) = \begin{pmatrix} 112 \\ 121 \\ 211 \end{pmatrix}\Big|_{1pt} \Rightarrow$ deg. $= 3 \;\Big|_{2pt}$

Second — : $= \begin{pmatrix} 221 \\ 212 \\ 122 \end{pmatrix}\Big|_{1pt} \Rightarrow$ deg. $= 3 \;\Big|_{1pt.}$

c) (A number of students read electrons with spin-½ as a statement on the magnitude (as intended) and their orientation being down (which was not intended.) This made the question a bit more difficult. To be fair, we have adapted the grading as :)

4pts - free
3pts - anti-symm. of total wavefunction
3pts - correct spatial & spin parts, which could be

$$\psi = \underbrace{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}_{\text{spin}} \cdot \underbrace{\psi_{n=1}(\vec{x}_1) \cdot \psi_{n=1}(\vec{x}_2)}_{\text{as above.}}$$

OR: $\psi = |\downarrow\downarrow\rangle \cdot \left(\psi_{n=2}(\vec{x}_1) \cdot \psi_{n=1}(\vec{x}_2) - \psi_{n=1}(\vec{x}_1) \cdot \psi_{n=2}(\vec{x}_2) \right)$

spin

where $\psi_{n=1}$ – as above

$\psi_{n=2}$ – $\sin\left(\frac{2\pi x}{a}\right) \cdot \sin\left(\frac{\pi y}{a}\right) \cdot \sin\left(\frac{\pi z}{a}\right)$

(example of $n=2$ state with $(n_x, n_y, n_z) = (2,1,1)$.

d) the energy of the ground state goes up (4pt) due to $e^- - e^-$ Repulsion: two electrons have equal charges & hence Repel ⇒ costs energy to bring them close. (3pt.) Same in Helium: ground state higher than in Bohr model (neglecting Coulomb e^-e^- interaction) (3pt.)

2a] $\psi(x,t) = a \cdot e^{-iE_0 t/\hbar} \cdot \psi_0 + b \cdot e^{-iE_2 t/\hbar} \cdot \psi_2$ (3pt.)

where $E_n = (n + 1/2)\hbar\omega$.

Not a stationary state (lin. comb. of stat. states).

~~3pt~~ 1pt

b) stand. dev. σ_x: $\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$ (2pt.)

$\langle x \rangle = 0$ as odd integrand. (2pt.) (4pt.)

$$\langle x^2 \rangle \equiv \int_{-\infty}^{+\infty} \psi^* \cdot x^2 \cdot \psi \, dx = \frac{5\hbar}{2m\omega}$$ 4pt.

which can be computed in two ways:

- explicitly with $\psi = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \cdot \frac{1}{2\sqrt{2}} \cdot 4\xi^2 \cdot e^{-\xi^2/2}$

  with $\xi^2 = \frac{m\omega}{\hbar} \cdot x^2$

  $\Rightarrow$ single Gaussian integral

- writing x^2 in terms of ladder operators $a_\pm$

c) $\sigma_p \neq 0$ as ψ_2 is not an eigenstate of p
   1pt 2pt.

but: σ_p time-indep. as ψ_2 is a stat. state.
   1 pt. 1 pt.

d) $[x^2, p^2] = 2\hbar^2 + 4i \cdot xp$. 5pt.

Relevant for Heisenberg as product of the standard dev. of two operators is bounded from below by their commutator:

$$\sigma_{x^2} \cdot \sigma_{p^2} \geq \frac{1}{2i} \cdot \langle [x^2, p^2] \rangle$$ 3pt.

RHS doesn't vanish $\Rightarrow$ cannot both be known with high accuracy. 2pt.

3a) pion decay $\Rightarrow$ singlet state $\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$ 2pt

$\Rightarrow$ measurement of one particle affects possible outcomes of measurement of other 3pt.

b) $\uparrow$ z $\qquad\qquad \swarrow$ x

$\Rightarrow$ $\left.\begin{matrix}\uparrow\uparrow \\ \uparrow\downarrow \\ \downarrow\uparrow \\ \downarrow\downarrow\end{matrix}\right\}$ all 25% probab. 5pt

c) $\uparrow \qquad\qquad \swarrow$

measures $\uparrow$: $\begin{pmatrix}1\\0\end{pmatrix}$ then is $\begin{pmatrix}0\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\cdot\left(|\uparrow\rangle_x \mp |\downarrow\rangle_x\right)$ 2pt 3pt.

where $\left.\begin{matrix}|\uparrow\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \\ |\downarrow\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}.\end{matrix}\right\}$ eigenstates of S_x.

d) $\psi_{e^+} = 0.6\,|\uparrow\rangle_\theta - 0.8\,|\downarrow\rangle_\theta$

$\Rightarrow$ $\left.\begin{matrix}|\uparrow\rangle & \text{prob. } 0.36 \\ |\downarrow\rangle & 0.64\end{matrix}\right\}$ $P = -0.28$ 3pt.

3pt

general: $P = -\cos\theta$ $\Rightarrow$ $\theta = \arccos(0.28)$

2pt. $\approx 74°$ 2pt