

QUANTUM PHYSICS I - Feb. 3, 2023

Write your name and student number on **all** sheets. There are 3 problems in this two-hour exam, with 90 points in total. You are allowed to use the Griffiths book for consultation.

Problem 1: INFINITE CUBIC WELL (all 6 = 36 points)

Consider a quantum system with a 1D potential energy that is infinite outside of the interval $[-a, a]$, and that is zero inside the well. Moreover, we will consider a wavefunction that is at $t = 0$ approximately given by a step function, equal to $-A$ in the left half and to $+A$ in the right half of the well (and zero outside of the well). The actual wavefunction is a smooth anti-symmetric function that approximates this step function.

- a) Normalise the wavefunction. To which relation between the interval length a and the wavefunction coefficient A does this lead?
- b) Every wavefunction is a linear combination of stationary states. Out of which stationary states does the above approximate step function consist?
- c) Does the approximate step function change over time? If so, indicate which of its properties change over time: piecewise constant, anti-symmetric, normalised. Briefly explain your answers.
- d) When measuring the total energy of this system, a number of possible values can be found. Which energies have non-vanishing probabilities? And do these probabilities change over time? Briefly explain your answers.
- e) Calculate the expectation values of x , x^2 and p (all at $t = 0$).
- f) Using the Heisenberg uncertainty principle, argue whether or not the expectation value for p^2 can be zero.

Problem 2: HARMONIC OSCILLATOR (all 6 = 30 points)

Consider a single particle in a 2D version of the harmonic oscillator system, with potential energy

$$V = \frac{1}{2}m\omega^2(x^2 + y^2). \quad (1)$$

- a) Can a particle in this system be in a quantum state where one knows the observables x and p_y with arbitrarily high precision? Briefly explain why this is (not) possible.
- b) Write down the full wavefunction $\psi(t, x, y)$, including time-dependence but not normalisation, for the first excited state (i.e. one above the ground state) of this system. (In case there are several, it suffices to write down one example.)

- c) Express the full Hamiltonian in polar coordinates r and θ . Hint: the Laplacian on a function f reads in 2D polar coordinates

$$\nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} f \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} f. \quad (2)$$

- d) Stationary states $\psi(x)$ can be split by separation of variables into $R(r)$ and $g(\theta)$. What are the possible stationary states for the $g(\theta)$ part, i.e. what are the angular eigenfunctions for this 2D system?
- e) To which angular eigenfunction (or linear combination of angular eigenfunctions) does your answer under b) correspond? Would you say this state carries angular momentum or not? Briefly explain your answer.

Problem 3: SPIN ADDITION (all 6 = 24 points)

Imagine that the headlines of tomorrow's news indicate the discovery of a new type of particle, the gravitino. In contrast to the known quarks and leptons, this would be a fermion with spin equal to $3/2$ (instead of $1/2$).

- a) What are the possible quantum numbers (s, m_s) of the spin configuration of this particle?
- b) Consider a combination of a gravitino particle with an electron. We are only interested in the spin configuration of this particle. What are the possible values of the total spin of this system?
- c) Do the wavefunctions of this system need to be symmetric or anti-symmetric under interchange of both particles, or is there no such requirement? Briefly explain your answer.
- d) The spin configurations of the separate two particles can be labelled by quantum numbers $(s^1, m_s^1; s^2, m_s^2)$, where 1 refers to the gravitino and 2 to the electron. Indicate how linear combinations of these are eigenstates of the total spin (and its z -component) of the system; in other words, how do these form spin ladders? You only have to indicate the general form of these linear combinations (i.e. which configurations are present in it), not the specific coefficients in it.