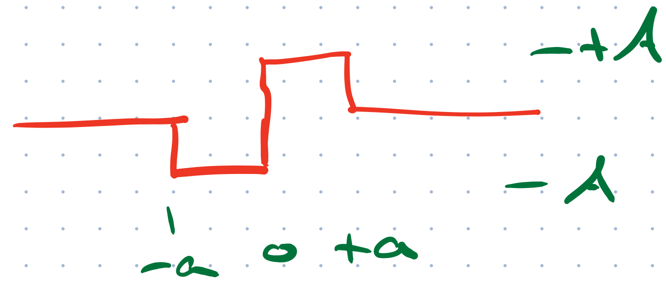
QP1 - Feb 3, 2023



1] a) Normalisation:

$$\int \psi^* \psi = 1$$

$$\Rightarrow 2 \cdot a \cdot |A|^2 = 1 \rightarrow A = \frac{1}{\sqrt{2a}}.$$

b) antisymm $\Rightarrow$ lin. comb. of all ODD stationary states.

$\sim$ only sin, no cos.

c) yes, changes over time (lin. comb. of stat. state $\neq$ stat. state itself).

piecewise constant – changes (otherwise, WF cannot change).

anti-symm - preserved
(as lin. comb. of AS states).

normalised - preserved
(preserved under time evol.
of Schr. eq.).

d) Possible energies :
all corresponding to odd excited
states $\Rightarrow$ $E_{2,4,6,8}$ . . . .

Prob. constant over time
(preserved under time evol.
under Schr. eq.)
(as stat. states have
overall time-dep.).

e) $\langle x \rangle = 0$
$\langle p \rangle = 0$) due to anti-symm. of integrand.

$$\langle x^2 \rangle = \int_{-a}^{+a} x^2 \cdot A^2$$

$$= \frac{1}{2a} \cdot \int_{-a}^{+a} x^2 = \frac{1}{6a} \cdot x^3 \Big|_{-a^3}^{a^3}$$

$$= \frac{1}{3} \cdot a^2 .$$

f) $\langle p^2 \rangle$ cannot be zero since Heisenberg:

$$\sigma_x \cdot \sigma_p \geq \frac{\hbar}{2} .$$

$$= \sqrt{\langle x^2 \rangle \langle p^2 \rangle}$$

2) a) Yes, as x and p_y commute

b) $$\psi_0(t,x,y) = \psi_0(x)\cdot\psi_1(y)\cdot e^{-iEt/\hbar}$$

$$E = 2\hbar\omega.$$

$$\psi_0(x) = A_0 \cdot e^{-\frac{m\omega}{2\hbar}\cdot x^2}$$

$$\psi_1(y) = A_1 \cdot e^{-\frac{m\omega}{2\hbar}\cdot x^2} \cdot i\cdot\omega\cdot\sqrt{2m}$$

c) $$H = -\frac{1}{2m}\cdot\nabla^2 + \frac{1}{2}m\omega^2 R^2$$

$$\nabla^2 = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2}{\partial\theta^2}$$

d) eigenfunctions: for $g(\theta)$

$g(\theta) = e^{im\theta}$ $\quad m$ integer.

e) ang. dependence:

$g \to \sin\theta$

lin. comb. of $e^{i\theta}$ and $e^{-i\theta}$

$\Rightarrow$ of ang. mom. with one unit (in clockwise / counter-direction)

3) a) $(s, m_s) = \left(\begin{array}{ll} \frac{3}{2}, & +\frac{3}{2} \\ \quad " & +\frac{1}{2}, \\ \quad " & -\frac{1}{2}, \\ \quad " & -\frac{3}{2} \end{array} \right)$

b) $S_1 = +\frac{3}{2}$ & $S_2 = \frac{1}{2}$

$\Rightarrow$ total $S = +2$ OR $+1$.

c) neither, as distinguishable particles.

d) $S = +2$: ladder with 5 states

$S = +1$ — 3 —.

in terms of (m_s^1, m_s^2)

$S=+2$ ladder:

$$(+3/2, +1/2)$$

$$(+3/2, -1/2) \; \& \; (+1/2, +1/2)$$

$$(1/2, -1/2) \; \& \; (-1/2, +1/2)$$

$$(-1/2, -1/2) \; \& \; (-3/2, +1/2)$$

$$(-3/2, -1/2).$$

$S=1$ ladder.