

$$\textcircled{1} \quad |\phi_L\rangle \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |\phi_R\rangle \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\langle \phi_L | \hat{H} | \phi_L \rangle = \langle \phi_R | \hat{H} | \phi_R \rangle = E \quad \langle \phi_L | \hat{H} | \phi_R \rangle = \text{c.c.} = T < 0$$

a)

$$\hat{H} \rightarrow \begin{pmatrix} E & T \\ T & E \end{pmatrix}$$

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$$(E - E)^2 - T^2 = 0$$

$$E - E = \pm T$$

$$\boxed{E = E \mp T}$$

eigenvalues

$$\begin{pmatrix} E & T \\ T & E \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = (E \mp T) \begin{pmatrix} a \\ b \end{pmatrix} \Rightarrow$$

$$Ea + Tb = (E \mp T)a$$

$$Ta + Eb = (E \mp T)b$$

$$a = \mp b$$

Eigenvalue

$$E + T$$

$$E - T$$

Eigen vector

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\text{Eigenstate } |\phi_g\rangle = \frac{1}{\sqrt{2}} (|\phi_L\rangle + |\phi_R\rangle) \quad \frac{1}{\sqrt{2}} (|\phi_L\rangle - |\phi_R\rangle) = |\phi_e\rangle$$

ground state

excited state

$$E + T < E - T \quad \text{for } T < 0$$

$$b) \hat{P} \equiv |\phi_L\rangle\langle\phi_R| + |\phi_R\rangle\langle\phi_L|$$

$$\hat{P}|\phi_g\rangle = \left(|\phi_L\rangle\langle\phi_R| + |\phi_R\rangle\langle\phi_L|\right) \frac{1}{\sqrt{2}} \left(|\phi_L\rangle + |\phi_R\rangle\right)$$

$$= \frac{1}{\sqrt{2}} |\phi_L\rangle + \frac{1}{\sqrt{2}} |\phi_R\rangle \Rightarrow \boxed{\hat{P}|\phi_g\rangle = |\phi_g\rangle}$$

$$\hat{P}|\phi_e\rangle = -\frac{1}{\sqrt{2}} |\phi_L\rangle + \frac{1}{\sqrt{2}} |\phi_R\rangle \Rightarrow \boxed{\hat{P}|\phi_e\rangle = -|\phi_e\rangle}$$

$|\phi_g\rangle$ and $|\phi_e\rangle$ are eigenstates of $\hat{P}$ (eigenr. ± 1).

$$\hat{P} \rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$[\hat{H}, \hat{P}] = 0$$

since they share the same basis.

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c) The state after the measurement is $|\phi_L\rangle \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

$$\langle \hat{A}(t) \rangle = \langle \phi_L(t) | \hat{A} | \phi_L(t) \rangle$$

$$|\phi_L\rangle = \frac{1}{\sqrt{2}} (|\phi_g\rangle + |\phi_e\rangle)$$

$$|\phi_L(t)\rangle = \frac{1}{\sqrt{2}} \left(e^{-iE_g t/\hbar} |\phi_g\rangle + e^{-iE_e t/\hbar} |\phi_e\rangle \right) \quad \text{with } E_g = E + T, \quad E_e = E - T$$

$$\hat{A} |\phi_g\rangle = -a |\phi_e\rangle \quad \hat{A} |\phi_e\rangle = -a |\phi_g\rangle$$

$$\langle \hat{A}(t) \rangle = \frac{1}{\sqrt{2}} \left[\langle \phi_g | e^{iE_g t/\hbar} + \langle \phi_e | e^{iE_e t/\hbar} \right] \hat{A} \frac{1}{\sqrt{2}} \left[e^{-iE_g t/\hbar} |\phi_g\rangle + e^{-iE_e t/\hbar} |\phi_e\rangle \right]$$

$$= -\frac{a}{2} \left[\langle \phi_g | e^{iE_g t/\hbar} + \langle \phi_e | e^{iE_e t/\hbar} \right] \left[e^{-iE_g t/\hbar} |\phi_e\rangle + e^{-iE_e t/\hbar} |\phi_g\rangle \right]$$

$$= -\frac{a}{2} \left(e^{-i(E_g - E_g) t/\hbar} + e^{i(E_e - E_g) t/\hbar} \right)$$

$$\boxed{\langle \hat{A}(t) \rangle = -a \cos \left[\frac{(E_e - E_g)}{\hbar} t \right] = -a \cos \left[-\frac{2T}{\hbar} t \right]}$$

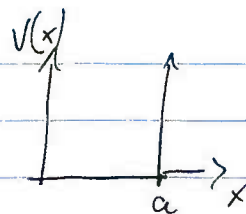
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d) Ground state: Both electrons at $|\phi_g\rangle$
at spin singlet state

$$|\text{ground}\rangle = (|\phi_g\rangle|\phi_g\rangle)|\chi_{\text{singlet}}\rangle$$

$$|\chi_{\text{singlet}}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$(2) \quad V(x) = \begin{cases} 0 & , \quad 0 \leq x < a \\ \infty & , \quad \text{otherwise} \end{cases}$$



$$a) \quad -\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} = E \psi \quad (\text{inside well})$$

$$\frac{d^2 \psi}{dx^2} = -\frac{2mE}{\hbar^2} \psi \Rightarrow \frac{d^2 \psi}{dx^2} = -k^2 \psi \quad k \equiv \frac{\sqrt{2mE}}{\hbar}$$

$$1 \quad \text{Solutions: } \psi(x) = A \sin(kx) + B \cos(kx)$$

$$2 \quad \text{Boundary cond. } \psi(x=0) = 0 \Rightarrow B = 0$$

$$\psi(x=a) = 0 \Rightarrow A \sin(ka) = 0$$

$$ka = n\pi \Rightarrow k = \frac{n\pi}{a}$$

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$$\boxed{\begin{aligned} \psi_n(x) &= A \sin\left(\frac{n\pi}{a} x\right) \\ E_n &= \frac{n^2 \hbar^2 \pi^2}{2m a^2} \end{aligned}}$$

$$\frac{n\pi}{a} = \frac{\sqrt{2mE}}{\hbar}$$

b) Ground state: $\psi_1 = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)$

3.5 $\langle x \rangle = \frac{a}{2}$ since the well is symmetric around $a/2$ with $|\psi_1|^2$ peaking at this value.

3.5 $\langle p \rangle = 0$ since particle is trapped in the well.
also $\int_0^a \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi x}{a}\right) dx = 0$

6 c) $\langle x \rangle$ and $\langle p \rangle$ do not change in time
since we are considering a stationary state.

d) $a \rightarrow 2a$
 $\psi'_n = \sqrt{\frac{2}{(2a)}} \sin\left(\frac{n\pi x}{(2a)}\right)$ 2.5

$E'_n = \frac{n^2 \pi^2 \hbar^2}{2m (2a)^2}$ 2.5

$$e) \quad \psi(x=0) = \begin{cases} \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right) & , \text{ for } 0 \leq x \leq a \\ 0 & , \text{ otherwise} \end{cases} \quad 3$$

$$\psi(x, t) = \sum_{n=1}^{\infty} c_n \psi_n' e^{-i\epsilon_n' t/\hbar} \quad 4 \text{ with } \psi_n' \text{ and } \epsilon_n' \text{ specified in (d).}$$

$$c_n = \int_{-\infty}^{\infty} \psi_n'^* \psi(x, t=0) dx$$

$$c_n = \int_0^a \left[\sqrt{\frac{2}{2a}} \sin\left(\frac{n\pi x}{2a}\right) \right] \left[\sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right) \right] dx$$

$$c_n = \int_0^a \frac{2}{a\sqrt{2}} \sin\left(\frac{n\pi x}{2a}\right) \sin\left(\frac{\pi x}{a}\right) dx \quad 3$$

$$(3) a) [L_z, x] = i\hbar y$$

$$\vec{L} = \vec{r} \times \vec{p} \rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix} \Rightarrow L_z = (x p_y - y p_x)$$

$$[L_z, x] = [x p_y, x] - [y p_x, x]$$

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$$= x p_y x - x x p_y - y p_x x + x y p_x$$

$$= x x p_y - x x p_y - y (p_x x - x p_x)$$

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$$= y (x p_x - p_x x) = y [x, p_x] \Rightarrow [L_z, x] = i\hbar y$$

$$b) \quad Y_1^0(\theta, \phi) = \left(\frac{3}{4\pi}\right)^{1/2} \cos\theta$$

$$L_{\pm} = L_x \pm i L_y \Rightarrow \text{find } Y_1^{\pm 1}$$

$$Y_1^{\pm 1} = L_{\pm} Y_1^0 \quad 1$$

$$L_{\pm} = -i\hbar \left\{ -\sin\phi \left(\frac{\partial}{\partial\theta}\right) - \cos\phi \cot\theta \left(\frac{\partial}{\partial\phi}\right) \pm i \cos\phi \left(\frac{\partial}{\partial\theta}\right) \mp i \sin\phi \cot\theta \left(\frac{\partial}{\partial\phi}\right) \right\}$$

$$L_{+} = i\hbar \left\{ [\sin\phi \mp i \cos\phi] \left(\frac{\partial}{\partial\theta}\right) + \cot\theta [\cos\phi \pm i \sin\phi] \left(\frac{\partial}{\partial\phi}\right) \right\}$$

$$L_{+} = \hbar \left\{ [i \sin\phi \pm \cos\phi] \left(\frac{\partial}{\partial\theta}\right) + \cot\theta [i \cos\phi \mp \sin\phi] \left(\frac{\partial}{\partial\phi}\right) \right\} \quad 2$$

$$L_{\pm} Y_1^0 = Y_1^{\pm 1} = -\hbar \left(\frac{3}{4\pi}\right)^{1/2} [i \sin\phi \pm \cos\phi] (\sin\theta)$$

$$= \mp \hbar \left(\frac{3}{4\pi}\right)^{1/2} e^{\pm i\phi} \sin\theta \quad 4$$

c) Possible values are $\pm \hbar/2$ 5 (2.5 if $\pm \hbar/2$)

$$\begin{aligned} d) \langle \Psi | \Psi \rangle &= \left[\cos(\theta/2) \langle \uparrow | + e^{-i\phi} \sin(\theta/2) \langle \downarrow | \right] \left[\cos(\theta/2) | \uparrow \rangle + e^{i\phi} \sin(\theta/2) | \downarrow \rangle \right] \\ &= \cos^2(\theta/2) \langle \uparrow | \uparrow \rangle + e^{-i\phi} \sin(\theta/2) \cos(\theta/2) \langle \downarrow | \uparrow \rangle + \\ &\quad + \cos(\theta/2) e^{i\phi} \sin(\theta/2) \langle \uparrow | \downarrow \rangle + e^{-i\phi} \sin^2(\theta/2) \langle \downarrow | \downarrow \rangle \\ &= \cos^2(\theta/2) + 0 + 0 + \sin^2(\theta/2) \end{aligned}$$

$$\langle \Psi | \Psi \rangle = \cos^2(\theta/2) + \sin^2(\theta/2) = 1 \quad // \quad 3$$

e) Possible outcomes are $\pm \hbar/2$ 2

Probabilities given by:

$$+\hbar/2: |\langle \uparrow | \Psi \rangle|^2 \quad \& \quad -\hbar/2: |\langle \downarrow | \Psi \rangle|^2$$

$$|\langle \uparrow | \Psi \rangle|^2 = \cos^2(\theta/2)$$

$$= \cos^2(30^\circ) = 3/4$$

$$|\langle \downarrow | \Psi \rangle|^2 = \sin^2(\theta/2)$$

$$= \sin^2(30^\circ) = 1/4$$

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