Problem 1: The 21 cm line

The 21 cm line is an important transition line in Astronomy. It enables us to understand the distribution of neutral Hydrogen in the universe.

Question 1.
Write the formula for the difference of two energy levels of an atom when emitting a photon. Write it in terms of λ, E_0 and E_1.

Solution 1.

$$E_1 - E_0 = h\frac{c}{\lambda}$$

Question 2.
Suppose the energy level of the lower state is $E_0 = -13.6$ eV, find E_1.

Solution 2.

$$E_1 = -13.599994 \text{ eV}$$

Question 3.
Compare this energy difference to the energy of a photon in the visible spectrum at 600 nm.

Solution 3.

$$E_1 = -11.53 \text{ eV}$$

The energy difference is 345000 times bigger compared to the 21 cm photon.

Problem 2: Luminosity of a star

Question 1.
Assuming that the sun can be approximated as a blackbody and that the temperature at the surface is 5000 degC, calculate the total luminosity of the sun using Stefan-Boltzmann law and the solar radius.

Solution 1.
First convert the temperature to Kelvin: 5000 Celsius becomes 5273 K.

$$L_1 = 4\pi R_s^2 \sigma T^4 = 2.671 \cdot 10^{26} \text{ W}$$

Question 2.
How many times more luminous is an O-class star with a surface temperature of 30000 K and a radius of 10 solar radii?

Solution 2.

$$L_2 = 4\pi 10 R_s^2 \sigma T^4 = 2.791 \cdot 10^{31} \text{ W}$$

$$\eta = \frac{L_2}{L_1} = 105000$$

The O star is 129615 more luminous.

Problem 3: Stellar Ages and Lifetimes

Before the discovery of thermonuclear reactions, it was commonly thought that gravity was the energy source of stars. If the gravitational energy of a given star of mass M_* and radius R_* is approximated by that of a body with a constant density, the energy emitted to space due to gravitational energy emitted since its formation may be approximated by the following expression;

$$\Delta U = \frac{3}{10}\frac{GM_*^2}{R_*}$$

Question 1.
By assuming that the Sun's luminosity $L_\odot$ has been constant since its formation, give a rough estimation of the age of the Sun assuming that gravity is the dominant source of energy for stars. This time is called the Kelvin - Helmholtz time.

Solution 1.
By assuming that the star's luminosity L_* has been constant since its formation, a characteristic time called the Kelvin - Helmholtz time can be defined as

$$\tau_{\text{KH}} = \frac{\Delta U}{L_*} = \frac{3}{10}\frac{GM_*^2}{L_* R_*}$$

For the Sun $\tau_{\text{KH}} \approx 10^7$yr, which gives a rough estimation of the age of the Sun assuming that gravity is the dominant source of energy for stars.

Since the age of the Sun is much larger than its Kelvin-Helmholtz time, gravity is not sufficient to generate the enormous amount of energy irradiated by the Sun since its formation. Another possible energy source in stars is chemical energy.

Question 2.
Assuming that 10 eV of energy per atom found in the Sun is emitted during some chemical reaction taking place there, calculate the time the Sun could shine at its present intensity if the only energy source was this chemical process. Assume that the Sun is composed of pure hydrogen.

Solution 2.
The mass of the Sun is found to be

$$M_\odot = 1.9891 \times 10^{33} \text{ g}$$

The atomic mass of hydrogen in grams

$$m_H = 1.674 \times 10^{-24} \text{ g}$$

The number of hydrogen atoms is therefore

$$N_H = \frac{M_\odot}{m_H} = 1.188 \times 10^{57}$$

The total energy that can be produced is

$$E = N_H \cdot 10 \text{ [eV]} = 1.188 \times 10^{58} \text{ eV}$$

The luminosity of the sun should be converted the right units

$$L_\odot = 3.8458 \text{ erg/s} = 2.4004 \times 10^{45} \text{ eV/s}$$

The time that the Sun could shine at its present intensity is therefore

$$t = \frac{E}{L_\odot} = \frac{1.188 \times 10^{58} \text{ [eV]}}{2.4004 \times 10^{45} \text{ [eV/s]}} = 4.949 \times 10^{12} \text{ s} \approx 157000 \text{ yrs}$$

Problem 4: Binary Systems

Question 1.
Two stars in a binary system are separated by an angular distance of 0.2” arcseconds. We would like to observe the sodium doublet, which is located at 589 nm for both stars. Use Rayleigh’s criterion to determine how big the minimum diameter D of the primary mirror should be to distinguish both stars.

Note. *Don’t forget to convert from arcseconds to radians.*

Solution 1.

$$D = \frac{1.22\lambda}{\theta} = \frac{3600 \cdot 180 \cdot 1.22 \cdot \lambda}{0.2\pi} = 0.74 \text{ m}$$

Another binary system is observed that consists of two stars orbiting a common centre of mass in a circular path. The orbital period is $P_{\text{orb}} = 12$ days. The first star, observed in the visible band, has a measured orbital speed of $v_1 = 34 \text{ km s}^{-1}$ determined using a method called Doppler spectroscopy. The second star, an X-ray pulsar, orbits around the centre of mass at a radius of $r_2 = 2.8 \times 10^{10}$ m.

Question 2.
What is the orbital radius of Star 1 about the centre of mass?

Solution 2.
The orbital velocity is given by

$$v_1 = \frac{2\pi r_1}{P_{\text{orb}}}$$

The orbital radius is therefore

$$
\begin{aligned}
r_1 &= \frac{P_{\text{orb}}\, v_1}{2\pi} \\
&= \frac{12 \cdot 24 \cdot 3600 \cdot 34 \times 10^3}{2\pi} \\
&\approx 5.610 \times 10^9 \text{ m}
\end{aligned}
$$

Question 3.
What is the total orbital separation, i.e.; the distance, between Star 1 and Star 2?

Solution 3.
The total orbital separation between the two stars

$$r = r_1 + r_2 \approx 3.4 \times 10^{10} \text{ m}$$

Question 4.
What is the total mass of the system?

Solution 4.

$$
\begin{aligned}
\frac{G(M_1 + M_2)}{r^3} &= \left(\frac{2\pi}{P_{\text{orb}}}\right)^2 \\
(M_1 + M_2) &= \left(\frac{2\pi}{P_{\text{orb}}}\right)^2 \frac{r^3}{G} \\
&= \left(\frac{2\pi}{12 \cdot 24 \cdot 3600}\right)^2 \frac{(3.361 \times 10^{10})^3}{6.6743 \times 10^{-11}} \\
&\approx 2.1 \cdot 10^{31} \text{ kg} \approx 10.5\ M_\odot
\end{aligned}
$$

Question 5.
What are the respective masses of Star 1 and Star 2?

Solution 5.
In every binary system:

$$M_1 r_1 = M_2 r_2$$

$$\frac{M_1}{M_2} = \frac{r_2}{r_1}$$

$$M_{tot} = M_1 + M_2$$

Combining these relations gives:

$$
\begin{aligned}
M_1 &= M_{tot} \cdot \frac{r_2}{r} \approx 8.6\ M_\odot \\
M_2 &\approx 1.9\ M_\odot
\end{aligned}
$$

In some other cases of close binaries, the two stars actually pass in front of each other, forming an eclipse that temporarily reduces the amount of light we see. Two examples of light curves are given in Figure 1.1 and Figure 1.2.

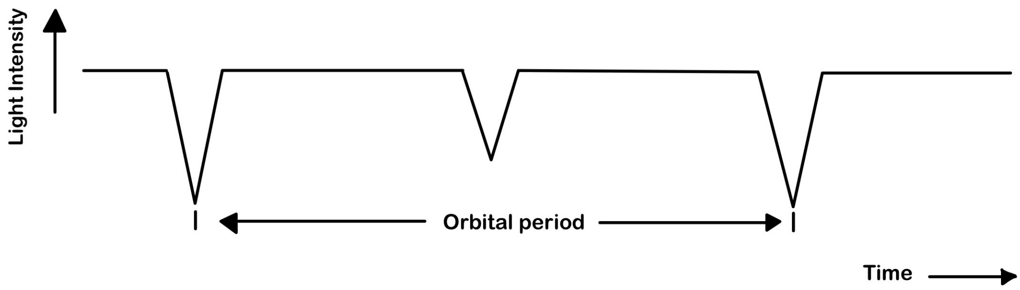

Figure 1.1

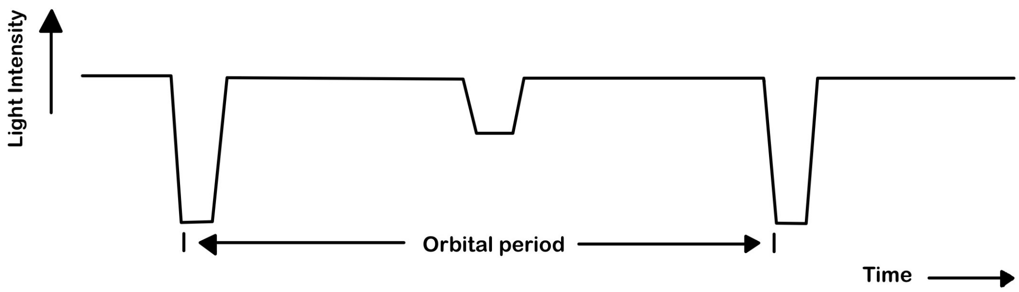

Figure 1.2

Question 6.
What can you say about the relative radii of the binary stars if the observed lightcurve is similar to Figure 1.1 and if the lightcurve is similar to Figure 1.2?

Solution 6.
The solution includes;

- If the lightcurve is similar to the lightcurve in Figure 1.1, it can be concluded that the binary stars have equal radii
- If the lightcurve is similar to the lightcurve in **??**, it can be concluded that the binary stars have different radii

Question 7.
The net amount of stellar surface is during an eclipse the same whether the smaller or bigger star is in front. So why is one of the eclipses deeper than the other? What quantity determines which of the eclipses will be deeper?

Solution 7.
The solution includes;

- The eclipse surface is the same, but the surface brightness of one star can be greater than the other.
- The surface brightness depends on temperature, so the eclipse of the hotter star will be deeper.

Problem 5: Night sky (from last-years midterm & exam)

Question 1.
You want to observe a cluster with a telescope. Its coordinates are: 12h 22.5m right ascension and +25 degrees declination. You use the Gratema telescope in Groningen at 53 degrees latitude. How many degrees do you have to tilt the telescope from the zenith to see the cluster?

(A) 28 degrees

(B) 62 degrees

(C) That depends on the time of the day

(D) That depends on the time of the year

(E) C and D

Solution 1.
E

Question 2.
We are observing a binary system with the Gratema telescope from Groningen (53 degrees latitude). What is their maximum altitude above the horizon at any point in time during the year if their coordinates are
RA: 06h34m 78s and DEC: 23d 00m 00s?

(A) 30 degrees

(B) 60 degrees

(C) 90 degrees

(D) They never get above the horizon

Solution 2.
B
Groningen is at 53 degrees latitude, meaning that the altitude of the NCP is 53 degrees at this location. The declination of 23 degrees means stars rotate at $90 - 23 = 67$ degrees around the NCP. Hence the maximum altitude is $180 - (53 + 67) = 60$ degrees.

Bonus problem: Stellarium

Use stellarium to answer the following questions:

Question 1.
Place your observer in Groningen.

a) Name three constellations that are circumpolar.

b) What is the declination right above you, called Zenith? How does it change with your position on Earth?

c) Go back to the year -12000. Is Polaris still the Pole Star? If not, which bright star is it? Explain this phenomena.

Solution 1.

a) Ursa minor, Cepheus, Cassiopeia, etc...

b) 53 degrees, changes with latitude

c) Vega, due to precession. Earth acts like a gyroscope, with the moon and sun exerting tidal forces.

Question 2.
Place your observer in Greenwich, United Kingdom.

a) What is the Right Ascension and Declination of the Sun on the 21st of March at mid-day (12:00) 2023? What is this event called?

b) What is the change in right ascension from the 21st to the 22nd of March at mid-day 2023? (23rd?, 24th?, etc...). Convert this to a time.

c) How much will it change over an entire year? Try to explain this phenomena.

Solution 2.

a) RA$=0$, DEC$=0$, Vernal equinox.

b) Change of 1 degree. This will be 4 minutes

c) It will change 24 hours, this is due to the fact that the earth rotates around the sun once every year. So when earth rotates once, it needs to rotate slightly longer for the sun to be back in the same position.

OR

The RA changes due to precession.