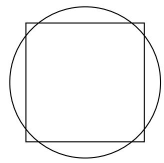
Electricity and Magnetism Test 4

7 May 2024, 18:30-20:30

The maximum score is 43 points. Good luck!

I. Short questions [19 points]

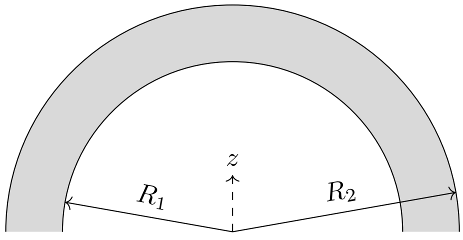
1. (5 points) Consider a battery with internal resistance through which a steady current flows. Make a drawing (no explanation needed) indicating:
   - The directions, rough relative magnitudes, and labels of the effective forces per unit positive moving charge *inside* the battery;
   - the direction in which the (positive) current flows;
   - which end of the battery has '+' printed on it.
2. (3 points) A circular wire loop lies in a uniform external magnetic field that points out of the page. You rearrange the wire into a square with an identical circumference, as drawn accurately in the figure on the right. Explain what the average direction of the current in the loop is during this procedure (as seen from above).



3. (2 points) Suppose $\mathbf{E} = y\widehat{\mathbf{x}} + xy\widehat{\mathbf{y}}$. Determine if the *magnetic* field is time-dependent.
4. (3 points) Explain why unplugging a device that draws a significant current is likely to generate sparks.
5. (3 points) We put a sinusoidal oscillating (alternating) current through the first coil of a transformer. The second coil will then also show an oscillating current. Determine by what fraction of a period the second coil's sinusoid lags behind the first. Ignore self-inductance.
6. (3 points) Can displacement currents produce a time-independent magnetic field? Explain using a Maxwell equation.

II. Hemispherical resistor [9 points]

Two concentric metal hemispheres (half-spheres) of radii R_1 and R_2 are connected by ohmic material of resistivity ρ. The inner and outer hemispheres are connected to the + and − of a battery, respectively, and a steady current flows. The figure below shows a cross-section of the setup; the shaded area is the ohmic material.



7. (3 points) Show/argue that the electric field in the ohmic material between the hemispheres is

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\widehat{\mathbf{r}} \tag{1}$$

where q_1 is the total charge on the inner hemisphere, and $\mathbf{r}$ is the distance from the origin of the spheres.

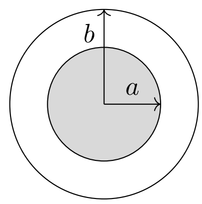
8. (3 points) Show that the current is radial and

$$I = \frac{q_1}{2\epsilon_0\rho}. \tag{2}$$

9. (3 points) Compute the resistance of this setup.

III. Coaxial cable [15 points]

Consider a long coaxial cable. A cross-section of the cable is shown in the figure on the right. Let s be the distance from the symmetry axis.



- A steady current I flows homogeneously (out of the paper) through a cylindrical solid wire that spans $s < a$.
- There is vacuum at $a < s < b$.
- At $s = b$, the current returns (into the paper) through a thin coaxial cylindrical shell.

10. (4 points) Argue that the magnetic field is:

$$\mathbf{B} = \frac{\mu_0 I}{2\pi}\hat{\phi}\begin{cases} s/a^2 & s < a \\ 1/s & a < s < b \\ 0 & s > b \end{cases}. \tag{3}$$

(Here $\widehat{\phi}$ rotates circumferentially around the symmetry axis, in the direction given by the right-hand rule.)

11. (4 points) Calculate the self-inductance of a length l of this cable.

For the rest of the problem, consider the time-dependent (alternating) current:

$$I(t) = I_0 \cos\omega t. \tag{4}$$

Here t indicates time and I_0 and ω are positive constants. Assume $\mathbf{B}$ is exactly given by equation 3, with the I of equation 4.

12. (2 points) Explain, using a Maxwell equation, why this assumption about $\mathbf{B}$ is reasonable only if ω is small.

13. (5 points) Calculate the induced/Faraday electric field in the region where $s < a$ (inside the inner wire), including its direction relative to the current. Ignore other contributions to the electric field.