Name: Student Number:

Mechanics and Relativity: M3

Mock exam
Duration: 120 mins

Before you start, read the following:

- There are 3 problems with subquestions, and you can earn 90 points in total. Your final grade is 1+(points)/10.
- Write your name and student number on all sheets.
- Make clear arguments and derivations and use correct notation. *Derive* means to start from first principles, and show all intermediate (mathematical) steps you used to get to your answer!
- Support your arguments by clear drawings where appropriate.
- Write your answers in the boxes provided. If you need more space, use the lined drafting paper.
- Generally use drafting paper for scratch work. Don't hand this in unless you ran out of space in the answer boxes.
- Write in a readable manner, illegible handwriting will not be graded.

Possibly relevant equations:

$$\vec{F} = m\vec{a}\,, \quad \vec{L} = \vec{r} \times \vec{p}, \quad \vec{\tau} = \vec{r} \times \vec{F}\,, \quad \vec{F}_{\text{centr}} = -m\vec{\omega} \times (\vec{\omega} \times \vec{r})\,, \quad \vec{F}_{\text{Cor}} = -2m\vec{\omega} \times \vec{v}\,, \quad \vec{F}_{\text{azim}} = -m(\frac{d\vec{\omega}}{dt}) \times \vec{r}\,,$$

and the Taylor expansion $(1+ax)^b \approx 1 + abx + \ldots$ at small x.

Question 1: Principal axes

(a) **(10 pts)** Give one of the equivalent definitions of principal axes; in other words, when is an axis principal?

(b) **(10 pts)** Imagine you are given an infinitely thin sheet of material that has an irregular planar shape (so not a circle or rectangle, but some shape without symmetry axis). Pick an arbitrary point on this arbitrary shape. How many principal axes w.r.t. this point lie in the plane of the sheet? Briefly explain your answer.

(c) **(10 pts)** Draw the principal axis of a tennis racket in order of ascending principal moments.

Question 2: Spinning chandelier

(a) **(15 pts)** Instead of a spinning top, consider a spinning chandelier that is suspended from the ceiling, and makes an angle θ with the vertical direction. Imagine it has principal moments $I_1 = I_2 \equiv I < I_3$, with the third principal axis along the spinning direction that is vertical when in rest. What is its precession frequency Ω in terms of its spinning frequency ω_3? You can use that the precession frequency Ω for a spinning top instead is

$$\Omega = \frac{I_3\omega_3}{2I\cos(\theta)}\left(1 - \sqrt{1 - \frac{4MIg\ell\cos(\theta)}{I_3^2\omega_3^2}}\right), \tag{1}$$

and it can help to think about the differences in geometrical configurations of both set-ups (that is, top and chandelier).

(b) **(15 pts)** Can the chandelier precess (at constant, non-vanishing angle θ) if it does not spin around its x_3 principal axis, that is when ω_3 vanishes? Briefly explain your answer using the no-spinning limit of the precession frequency (if you did not find an answer under a, use the above expression for the spinning top instead). If your answer is yes: give the precession frequency.

Question 3: Tidal forces

(a) **(15 pts)** Suppose we would consider a drop of water on Earth that is located along the axis of the Earth and the Moon. Derive what the tidal force (due to the Moon and as measured in the Earth's frame) on this drop of water would be. You can use the approximation of three point objects, $m_{1,2,3}$, on a straight line, with a distance x between m_2 (Earth) and m_1 (drop of water) that is much smaller than the distance R between the Earth and the moon (m_3). Does the tidal force point away or towards the Earth?

(b) **(15 pts)** Next, consider an imaginary Universe where the attraction between these celestial bodies is not due to gravity, with its $1/r^2$ fall-off, but instead the Earth and the Moon experience an attractive force that is of the form of Hooke's law, with $\vec{F} = -k\vec{r}$. In which direction does the tidal force point in such a Universe? Briefly explain your answer, based on physical reasoning / general considerations, not necessarily the full calculation.