Name: Student Number:

Mechanics and Relativity: M resit

February 15, 2024, Aletta Jacobshal
Duration: 120 mins

Before you start, read the following:

- There are 3 problems with subquestions, and you can earn 90 points in total. Your final grade is 1+(points)/10.
- Write your name and student number on all sheets.
- Make clear arguments and derivations and use correct notation. *Derive* means to start from first principles, and show all intermediate (mathematical) steps you used to get to your answer!
- Support your arguments by clear drawings where appropriate.
- Write your answers in the boxes provided. If you need more space, use the lined drafting paper.
- Generally use drafting paper for scratch work. Don't hand this in unless you ran out of space in the answer boxes.
- Write in a readable manner, illegible handwriting will not be graded.

Possibly relevant equations:

$$\vec{F} = m\vec{a}\,, \quad K = \tfrac{1}{2}mv^2\,, \quad V = \tfrac{1}{2}kx^2\,, \quad I = \int x^2 dm\,, \quad \tau = I\alpha\,, \quad \vec{F}_{\text{Cor}} = -2m\vec{\omega}\times\vec{v}\,,$$

and the Taylor expansions at small x:

$$(1+ax)^b \approx 1 + abx + \dots\,, \quad \sin(x) \approx x + \dots \quad \cos(x) \approx 1 - \tfrac{1}{2}x^2 + \dots\,. \tag{1}$$

Question 1: Harmonic oscillator

Consider a cart of mass m that can move back and forth on an air rail, such that you can ignore friction in this question. It is attached to a wall via a spring with spring constant k. The resulting oscillation has an eigenfrequency given by $\omega^2 = k/m$.

(a) **(10 pts)** Show with an explicit calculation that the total energy, consisting of kinetic and potential energy, is constant throughout this motion.

(b) **(10 pts)** Now add a driving force of the form $F_0 \cos(\omega_d t)$. What is the amplitude of the resulting driven motion for $\omega_d = \frac{1}{2}\omega$? You can ignore the homogeneous solution, and only focus on the particular solution in this case.

(c) **(10 pts)** What is the phase difference between the driving force and the driven oscillation when $\omega_d < \omega$? And what is it when $\omega_d > \omega$? Remember that you can ignore friction in this exercise.

Question 2: Moment of inertia and angular acceleration

Consider a stick of length L, mass M and uniform density $\rho = M/L$ that is fixed by a pivot point at a distance ℓ from its center of mass; as seen from the pivot point, the upper and lower parts therefore have lengths $L/2 \pm \ell$, see the picture. In this exercise you can restrict to planar motion, with only two spatial coordinates and one angle θ with the vertical direction.

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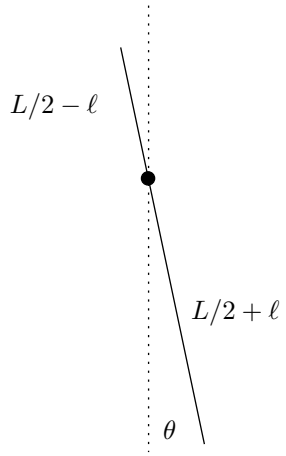

(a) **(15 pts)** Calculate the moment of inertia of the stick with respect to the pivot point.

(b) **(15 pts)** Calculate the magnitude of the torque due to gravity as a function of ℓ and the angle θ. Moreover, calculate the angular frequency ω (as a function of ℓ and of the moment of inertia I) of the resulting oscillatory motion of the stick under the influence of gravity. You can use the approximation that the angle θ is small.

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Question 3: Foucault's pendulum

Imagine a pendulum consisting of a heavy mass M hanging from a rope of length L at the North pole. For small oscillations, the time it takes for the pendulum to swing from left to right under the influence of gravity is $T = \pi\sqrt{L/g}$. The pendulum is so long that this takes four minutes. Use coordinates (x, y) for the surface at the North pole.

(a) **(10 pts)** The Coriolis force causes a small deflection what will make the pendulum precess. Consider a single swing from left to right, along the x direction: $x(t) = -x_0 \cos(\pi t/T)$. During this motion, what is the magnitude of the Coriolis force as a function of time? Sketch the motion that this leads to in the (x, y) plane, as the pendulum swings from left to right.

(b) **(20 pts)** As the pendulum swings from left to right along the x-axis, what is the velocity and the position (as a function of time) along the y-axis induced by the Coriolis force? How large is the deflection along the y-axis that this leads to during one swing from left to right? By how many degrees has the pendulum precessed during these four minutes? Remember that $\omega = 7{\cdot}10^{-5}$ 1/s, take x_0 to be 1 m and only calculate to one significant digit.