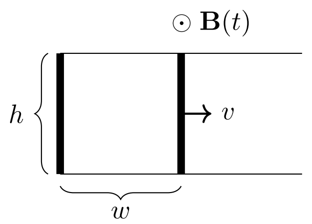
Electricity and Magnetism Test 4

9 May 2025, 18:30-20:30

- You may use your double-sided A4 cheat sheet, the provided formula sheet, and a calculator.
- Please leave some margins for grading, and do *not* use the white scratch paper for your final answers.
- Clearly indicate directions of vector quantities and shapes/locations of surfaces and loops of integration.
- The maximum score is 36.5 points. Good luck!

Short questions [13 points]

1. Two circular wire loops are placed nearby each other in such a way that they have zero mutual inductance.
   (a) (1 point) Define mutual inductance.
   (b) (1 point) Clearly sketch/indicate a possible arrangement of the loops. No explanation needed.
2. (5 points) Consider a battery that is *being recharged*. It has internal resistance, and a steady current flows. Make a drawing (no explanation needed) indicating:
   - The directions, rough relative magnitudes, and labels of the effective forces per unit positive moving charge *inside* the battery;
   - the direction in which the current flows;
   - which end of the battery has '+' printed on it.
3. (3 points) A bar of height h moves with constant speed v on rails to the right. At time $t = 0$ it is a distance w from a fixed bar on the left. The bars and rails are conductive. There is a uniform $\mathbf{B}$ with constant direction out of the page, whose magnitude $B(t)$ is time-dependent with $B(0) = B_0 \neq 0$. Find the $B(t)$ so that *zero current* is observed.



4. (3 points) A magnetic field changes linearly in time, so that $\frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$. Show that the displacement current has no curl.

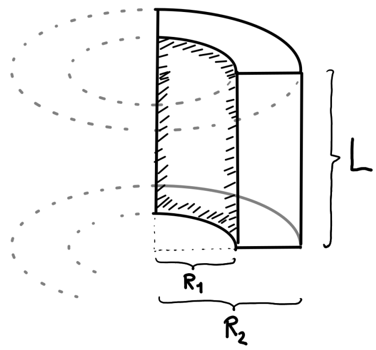
Quarter-cylinder resistor [9.5 points]

5. (2 points) Show that the electric field of an infinite static line of charge density λ is:

$$\mathbf{E} = \frac{\lambda}{2\pi\epsilon_0 s}\hat{\mathbf{s}}, \tag{2}$$

where s is the radial distance to the line.

Now consider the resistor shown on the right, consisting of a quarter-cylinder of ohmic material with conductivity σ and length L. The inner and outer curved surfaces (with radii R_1 and R_2, respectively) are the metal terminals/leads/electrodes. The electric field inside the resistor is the same as eq. 2, as we can verify this by checking three necessary and sufficient conditions (besides the fact that eq. 2 is an electrostatic field).

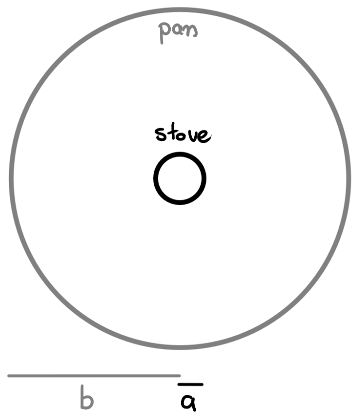


6. (1.5 points) State these three conditions on $\mathbf{E}$. You do not need to explain or prove them.
7. (6 points) Compute the resistance of this resistor. Explain your work.

Questions continue on the other side.

Induction stove [9 points]

Consider a simple model of a pan on an induction stove, sketched from above on the right.



We model the pan and stove each as a single wire loop, concentric in the same plane. The stove radius a is much smaller than the pan radius b, and the pan has a resistance R (large enough that any induced current in the pan does not significantly change the magnetic field). A counterclockwise current $I_s(t) = I_0 \cos(\omega t)$ flows through the *stove*, where I_0 and ω are positive constants.

In this problem, we will assume that Faraday quasistatics holds.

8. (2 points) What does that mean, and what (qualitatively) does this require for ω?

9. (2 points) At $t = \pi/\omega$, briefly explain the direction (if any) of the induced current in the *pan*.

10. (5 points) Find the induced current $I_p(t)$ through the *pan*. Hints:

    - If a loop of radius r in the xy plane carries a counterclockwise current I, the magnetic field at its *center* is:
    $$\mathbf{B} = \frac{\mu_0 I}{2r}\hat{\mathbf{z}}. \tag{3}$$
    - Be careful, the pan is the big loop! How can you still use the hint above and $a \ll b$?

Spinning cylinder [5 points]

An infinite cylinder of radius R carries a uniform surface charge σ. Although it has negligible mass, significant work is still needed to set it spinning around its axis.

11. (1 point) What opposes the acceleration of the cylinder? Answer in a few words (a sentence at most), but be specific.

12. (4 points) Find the minimum required work, per unit length of the cylinder, to set it spinning at an angular velocity of ω radians/second. Hint: a spinning charged cylinder is equivalent to...

This concludes the test. When you are finished, please:

- Write your **name and student number on every sheet**!
- If you used two sheets, mark them 'Sheet 1/2' and 'Sheet 2/2'. When you hand them in, bind them with **two paperclips** on opposite sides.
- Feed your solutions to the wooden box. Not in the box = not graded.
- Return your formula sheet and *unused* paper. Take this question paper, your cheat sheet, and *used* scratch paper home.