Electricity and Magnetism Test 4

9 May 2025, 18:30-20:30

- You may use your double-sided A4 cheat sheet, the provided formula sheet, and a calculator.
- Please leave some margins for grading, and do *not* use the white scratch paper for your final answers.
- Clearly indicate directions of vector quantities and shapes/locations of surfaces and loops of integration.
- The maximum score is 36.5 points. Good luck!

Solution: If you handed in the test, your grade will be:

$$\min\left(\ 10,\ \ 1 + 9 \cdot \frac{\text{Total points}}{33.5} + \text{Tutorial bonus (if applicable)}\right).$$

Note that 3 points were shifted to bonus to account for a small typo in the spinning cylinder question, and the general difficulty/length of the exam.

Instructions for grading TAs:

- If several students make a similar mistake, please write that down AND agree on a consistent way to score it!
- Only award full integer points equal or larger than zero. Exception: sometimes the rubric says half point may be awarded for a speciftic circumstance. If you think we need a new case for 0.5pt, agree with other TAs on a rule and write that down (so we can update the rubric).
- If the question asks for an explanation, calculation, determination, argumentation, etc., award no points if this is missing or clearly incorrect / incoherent.
- Subtract points for each mistake only once, unless the error substantially simplifies or alters the rest of the problem.
- Pay close attention to answers for 'show that ...' questions. Making two mistakes that miraculously cancel each other should be awarded *fewer* points than making one mistake and not reaching the result.

Short questions [13 points]

1. Two circular wire loops are placed nearby each other in such a way that they have zero mutual inductance.
   (a) (1 point) Define mutual inductance.
   (b) (1 point) Clearly sketch/indicate a possible arrangement of the loops. No explanation needed.

Solution: Mutual inductance is

$$M_{12} = \Phi_1 / I_2 \tag{1}$$

where Φ_1 is the magnetic flux through loop 1 caused by the current I_2 through loop 2. [1pt]

An arrangement with zero mutual inductance has one loop at right angles around the other (example c in tutorial 20). [1pt] (In this way there is zero magnetic flux through the second loop if there is a current through the other.)

Common mistake: "M is the ratio between the current induced in one loop by the flux of another loop." No, the induced current's magnitude depends also on the resistance of the loop; moreover 'flux of the other loop' is ill-defined.

2. (5 points) Consider a battery that is *being recharged*. It has internal resistance, and a steady current flows. Make a drawing (no explanation needed) indicating:

- The directions, rough relative magnitudes, and labels of the effective forces per unit positive moving charge *inside* the battery;
- the direction in which the current flows;
- which end of the battery has '+' printed on it.

Solution: Same as the electric motor figure in the lecture 18 notes.

One point each for:

- $\mathbf{f}_\rho$ / resistive force drawn and directed opposite to the current
- $\mathbf{f}_s$ / source force / battery force drawn and directed *opposite to* the current (the battery is charging, not helping the current)
- $\mathbf{E}$ / $-\nabla V$ / electric force / electrostatic force drawn and directed opposite to $\mathbf{f}_s$
- Forces are in equilibrium (i.e. $-\nabla V$ is roughly as large as $\mathbf{f}_s + \mathbf{f}_\rho$). Do not award if no forces are drawn at all.
- The current flows from + to - (unlike a regular battery).

Subtract one point for each other, spurious force drawn.

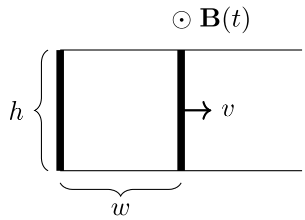
If someone draws a tiny arrow for gravity AND makes a correct note that gravity does not scale with charge: no penalty. If such a note is missing, subtract one point (the question asks for 'forces *per* unit positive charge').

The diagram for a 'super-short-circuited' battery (f_s and $-\nabla V$ working together against the resistive force) is worth 3.5 points: it would be 3 if we strictly apply the rubric above, but the half point is awarded in consideration of the fact that it can be remedied just by flipping the direction of f_s (and adjusting magnitudes).

If someone omits the resistive force, but the diagram clearly implies a net force directed opposite to the current AND there is a correct note about resistive processes happening (e.g. 'occasional jerky forces from collisions'), no penalty.

Common mistake: forgetting the resistive force (and adding no note about it).

3. (3 points) A bar of height h moves with constant speed v on rails to the right. At time $t = 0$ it is a distance w from a fixed bar on the left. The bars and rails are conductive. There is a uniform $\mathbf{B}$ with constant direction out of the page, whose magnitude $B(t)$ is time-dependent with $B(0) = B_0 \neq 0$. Find the $B(t)$ so that *zero current* is observed.



Solution: To have zero current we must have zero emf (Faraday and motional parts combined). That means there must be a constant flux $\Phi = B \cdot A$ through the loop [1pt].

The area of the loop obeys $A(t) = h \times (w + vt)$, so we need $B(t) \propto \frac{1}{w+vt}$ (only way to remove the time dependence of the flux). [1pt]

To fix the constant, require $B(0) = B_0$, so

$$B(t) = \frac{B_0}{1 + \frac{v}{w}t}. \quad \text{[1pt]}$$

Alternative: Though it is considerably more complicated and unnecessary, you could also approach the problem as a differential equation. After arguing, as above, that the magnetic flux is constant, solve

$$0 = \frac{d\Phi}{dt} = B'(t)A(t) + B(t)A'(t) \tag{2}$$

with $A = h(w + vt)$ and $dA/dt = hv$, giving

$$B'(t) = -B(t)\frac{hv}{h(w+vt)} = -B(t)\frac{v}{w+vt} \tag{3}$$

Now solve this as usual:

$$\begin{aligned}
\int \frac{dB}{B} &= \int \frac{vdt}{w+vt} \\
\ln B(t) &= -\ln\left(1 + \frac{v}{w}t\right) + C \\
B(t) &= \frac{e^C}{1 + \frac{v}{w}t}
\end{aligned}$$

and fix the proportionality constant by requiring $B(0) = B_0$ as above.

Common mistake: Not realizing that the flux needs to remain constant, then half-assing the differential equation (for example, not pulling in a minus sign into the logarithm). Not applying the initial condition $B(0) = B_0$.

4. (3 points) A magnetic field changes linearly in time, so that $\frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$. Show that the displacement current has no curl.

Solution: The displacement current (density) is $\mathbf{J}_D = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$ [1pt].

Taking the curl and using Faraday's law ($\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$):

$$\begin{aligned}
\nabla \times \mathbf{J}_D &= \nabla \times \left(\epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right) \\
&= \epsilon_0 \frac{\partial}{\partial t}(\nabla \times \mathbf{E}) \quad \text{[1pt]} \\
&= -\epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0 \quad \text{[1pt]}
\end{aligned}$$

Remark: The significance of this is that the displacement current is *unobservable*, meaning it does not affect the magnetic field *at all*! (Proof in the next paragraph.) Each little piece of $\mathbf{J}_D$ still affects $\mathbf{B}$, but the total contributions of displacement current cancel out. This is the source of the strange observation in tutorial 21 that you could find the magnetic field in a charging capacitor by considering the conduction current alone. To observe effects of displacement currents, you need rapidly changing fields (with second time derivatives), such as in electromagnetic waves.

Let's prove that a curl-free (displacement) current is unobservable. For simplicity, consider a situation without conduction currents, so the Ampere-Maxwell law becomes $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}_D$. But $\mathbf{J}_D$ has no curl here, so

$$0 = \nabla \times \nabla \times \mathbf{B} = \nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = -\nabla^2 \mathbf{B},$$

where we used identity 11 from the formula sheet and Noname's law. This is three copies of Laplace's equation, and from electrostatics you know that the only solution to this that vanishes at infinity is $\mathbf{B} = 0$ everywhere.

Common mistakes: adding μ_0 to the definition of displacement current, or misnaming Faraday's law as some other law (each minus half a point).

Quarter-cylinder resistor [9.5 points]

5. (2 points) Show that the electric field of an infinite static line of charge density λ is:

$$\mathbf{E} = \frac{\lambda}{2\pi\epsilon_0 s}\hat{\mathbf{s}}, \tag{4}$$

where s is the radial distance to the line.

Solution: Straightforward part 1 question:

- The field must be radial by symmetry.
- Take a cylindrical Gaussian surface of length L centered on the wire. (Ideally, draw it!)
- The flux through the caps is zero, so the total flux is only that through the curved surface.
- This is $\Phi_E = E_s A = E \cdot 2\pi s L$.
- The charge inside is $Q = \lambda L$.
- $Q = \Phi_E$ by Gauss' law, so $\lambda L = E_s \cdot 2\pi s L$, from which the result follows.

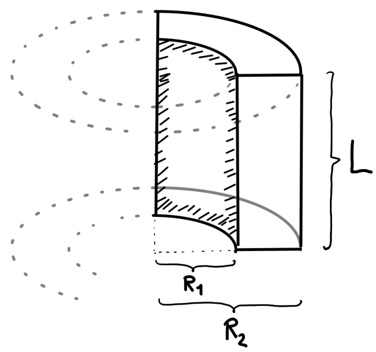
For a full score, you *must* mention that the field is radial by symmetry *or* that the flux through the caps of the surface is zero. If both are omitted, but you still indicated the integration surface clearly, you lose only half a point. If both are omitted *and* you don't clearly indicate the integration surface, you lose a full point.

For example, just writing $Q = \lambda L = EA = E \cdot 2\pi s L$ and solving for E would get you one point.

Common mistakes: Some students used Amperian loops (?!), or even used Coulomb's law (possible, but then you have to integrate carefully). Several students lost points for not indicating the integration surface. A few did use Gauss law with a correct surface, but said the enclosed charge is just λ rather than λL.

Now consider the resistor shown on the right, consisting of a quarter-cylinder of ohmic material with conductivity σ and length L. The inner and outer curved surfaces (with radii R_1 and R_2, respectively) are the metal terminals/leads/electrodes. The electric field inside the resistor is the same as eq. 4, as we can verify this by checking three necessary and sufficient conditions (besides the fact that eq. 4 is an electrostatic field).

6. (1.5 points) State these three conditions on $\mathbf{E}$. You do not need to explain or prove them.



Solution:

- The field has no divergence (except at the line charge, but that is outside the resistor). (So there is no net charge density inside the resistor.) [0.5pt]
- The field is perpendicular to the leads (so they are at equipotential) [0.5pt].
- The field is parallel to the other walls. [0.5pt] (So the current density is too, and the walls don't forever accumulate charge.)

Note: the fact that the field is cylindrically symmetric is *not* one of the conditions; the resistor is clearly *not* spherically symmetric. Award no points for $\nabla \times \mathbf{E} = 0$, a Maxwell equation, etc.; the question asked for conditions *besides* the fact that eq. 4 is an electrostatic field.

Common mistakes: Being too ambiguous (and hence sometimes simply wrong) when stating the conditions on E. For example, stating that the whole system is an equipotential (which would mean no driving force here, and hence no current).

7. (6 points) Compute the resistance of this resistor. Explain your work.

Questions continue on the other side.

Solution: By Ohm's law, the current density inside the resistor is $\mathbf{J} = \sigma \mathbf{E}$. To find the total current, integrate over a radial surface at distance s, which will have area $\pi s L/2$ (full cylindrical shell would be $2\pi s L$). Thus

$$I = \int \mathbf{J} \cdot d\mathbf{A} = \int \sigma \mathbf{E} \cdot d\mathbf{A} = \sigma E_s \pi s L/2 = \frac{\sigma \lambda L}{4\epsilon_0}.$$

directed radially outward.

To find the potential difference, integrate the electric field along a radial path:

$$\Delta V = \int_{R_1}^{R_2} \mathbf{E} \cdot d\ell = \int_{R_1}^{R_2} E_s ds = \frac{\lambda}{2\pi\epsilon_0} \int_{R_1}^{R_2} \frac{ds}{s} = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{R_2}{R_1}\right).$$

Thus the resistance is

$$R = \frac{\Delta V}{I} = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{R_2}{R_1}\right) \cdot \frac{4\epsilon_0}{\sigma\lambda L} = \frac{2}{\sigma\pi L} \ln\left(\frac{R_2}{R_1}\right).$$

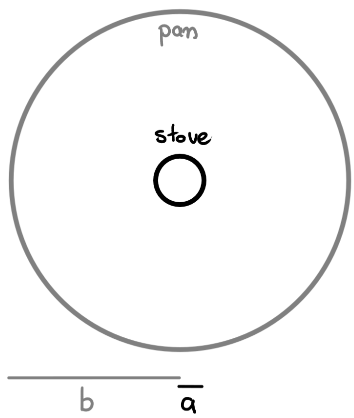
Scoring guide:

- Three points for a correctly explained derivation of the current. Substract a point if the area is not correct (probably you'll see $\pi s L/4$ or $\pi s^2 L/4$), and also substract a point if the method appears correct but the explanation is seriously deficient (not clear what the integration surface is, for example).
- Two points for a correct potential difference. Substract a point for integration/math error; subtract another point for not making the method and path both clear.
- Final point for a correct conclusion. An consistent result that follows from an incorrect ΔV and/or I also gets this point if one of these two conditions is met: (a) the answer is physically reasonable, meaning it is positive, inversely proportional to σ, and involves purely geometric quantities (so not λ, ΔV, I, etc), *or* (b) there is a clear note that this conclusion cannot be right. (E.g. "'λ is still in there, that's wrong since resistance should be purely geometric.")

  In particular, you don't get the final point just for writing down $R = \Delta V/I$.

Common mistakes: Forgetting the factor 4 from dealing with a quarter cylinder. Integrating over wrong surfaces (e.g. over ds) or not indicating what surface is used. Getting an answer that makes no sense (e.g. has wrong units) due to calculation errors, then not realizing it. Sign errors in the potential calculation.

Induction stove [9 points]



Consider a simple model of a pan on an induction stove, sketched from above on the right.

We model the pan and stove each as a single wire loop, concentric in the same plane. The stove radius a is much smaller than the pan radius b, and the pan has a resistance R (large enough that any induced current in the pan does not significantly change the magnetic field). A counterclockwise current $I_s(t) = I_0 \cos(\omega t)$ flows through the *stove*, where I_0 and ω are positive constants.

In this problem, we will assume that Faraday quasistatics holds.

8. (2 points) What does that mean, and what (qualitatively) does this require for ω?

Solution: If Faraday quasistatics hold, displacement currents are negligible / changes in electric fields do not appreciably affect the magnetic field. [1pt]

To achieve this, ω must be small, so the fields do not vary too quickly. [1pt]

(Given the sinusoidal current, $B \propto I$, $E \propto \frac{\partial B}{\partial t} \propto \omega$ so $J_D = \epsilon_0 \mu_0 \frac{\partial E}{\partial t} \propto \omega^2$.)

You get 1.5 points for a correct explanation of quasistatics followed by a bare 'so ω must be small', to recognize that the extra required explanation is tiny (just that ω controls the relevant rate of change).

Common mistakes:

- "Quasistatics means $\frac{\partial B}{\partial t}$ is small so the electric field is just that of electrostatics." No, that would mean there is no Faraday induction at all!
- Similarly, setting $\omega = 0$ would mean there is no changing current and therefore no induction. That's statics, not quasistatics; and your induction stove won't work without Faraday's law.

9. (2 points) At $t = \pi/\omega$, briefly explain the direction (if any) of the induced current in the *pan*.

Solution: The stove current is at an extremum at this time, $dI/dt = 0$ [1pt].

(The stove current is the only source of magnetic field: we assumed displacement currents and the effect of the current through the pan is negligible.)

Thus $dB/dt = 0$ everywhere, meaning there is no induced emf and therefore no current in the pan. [1pt]

Common mistakes:

- "The induced current is counterclockwise because the source current at this time is clockwise since $\cos\pi = -1$" (or arriving at this following some right-hand rule drama). No, induction responds to *changes* in magnetic fields and therefore *changes* in currents. Zero points.
- Just saying it's zero without any argument – sorry, no points. You need to explain why it's zero.
- '$\cos\pi = 0$ so no induced current': zero points, you made a double error (actually $\cos\pi = -1$, and it's the rate of change of the induced current that matters).
- A few students realized the source current was at an extremum but still argued for a nonzero induced current. For example, you could invoke lags from self-inductance – that's clever, but unfortuately the question said to ignore the effect of the current through the pan on the magnetic field. One point.

10. (5 points) Find the induced current $I_p(t)$ through the *pan*. Hints:

- If a loop of radius r in the xy plane carries a counterclockwise current I, the magnetic field at its *center* is:

$$\mathbf{B} = \frac{\mu_0 I}{2r}\hat{\mathbf{z}}. \tag{5}$$

- Be careful, the pan is the big loop! How can you still use the hint above and $a \ll b$?

Solution: We can use the symmetry property of mutual inductance [1pt]. First, assume that a current I_p flows through the *pan*, and consider the flux Φ_s through the stove. Because $a \ll b$, we can assume the field is uniform and equal to its value at the center. [1pt] Thus:

$$M = \frac{\Phi_s}{I_p} = \frac{BA_s}{I_P} = \frac{\frac{\mu_0 I_p}{2b}\pi a^2}{I_p} = \frac{\mu_0 \pi a^2}{2b} \quad \text{[1pt]}. \tag{6}$$

Now return to the question at hand. If a current I_s flows through the stove, the emf induced in the pan is

$$\mathcal{E} = -M\frac{dI_s}{dt} = -\frac{\mu_0 \pi a^2}{2b}\frac{dI_s}{dt} = -\frac{\mu_0 \pi a^2}{2b}(-\omega I_0 \sin(\omega t)) = \frac{\mu_0 \pi a^2 \omega I_0}{2b}\sin(\omega t) \quad [1\text{pt}]. \tag{7}$$

and given Ohms' law, $I_p = \mathcal{E}/R$ we find the result:

$$I_p(t) = \frac{\mu_0 \pi a^2 \omega I_0}{2Rb}\sin(\omega t). \quad [1\text{pt}] \tag{8}$$

Scoring aid: The first two points are for the mutual inductance part of the argument. You get the first point for just mentioning M (unless you really just called it M_{12}) or somehow incorporating it in the answer. The final three points are for a correct conclusion from here – but it is difficult to set up a coherent argument without the mutual inductance insight.

Common mistakes:

- Some students changed the question to assume the current I_s is really in the pan, computing the induced current in the stove, and calling that I_p. This results in having a and b interchanged:
$$I_p(t) = \frac{\mu_0 \pi b^2 \omega I_0}{2Ra}\sin(\omega t). \tag{9}$$
This gets a maximum of 3 points, since the first part on mutual inductance is missing.
- If you do the above mistake and *also* erroneously interchange a and b, you spuriously get the correct answer! Unfortunately, you only earn 2 points, due to the additional mistake on top of the common mistake.
- Several students used the same radius for both the flux area and the magnetic field, essentially computing a self-inductance.
- A few students left r in the final answer, or even integrated over it for some reason. (Personally I suggest differentiating when you really don't understand something.)

Alternative solution: You could also disregard the hint and use the following method, which requires more magnetostatics. Approximate the stove loop as a magnetic dipole, whose field is given by (equation 5.88 fourth edition):

$$\mathbf{B} = \frac{\mu_0 m}{4\pi r^3}\left(2\cos(\theta)\hat{\mathbf{r}} + \sin(\theta)\hat{\theta}\right). \tag{10}$$

The magnetic dipole moment $m = I_s A_s = I_s \pi a^2$ in the z-direction. In the plane of the loop, $\theta = \pi/2$ so $\cos\theta = 0$, $\sin\theta = 1$, and $\hat{\theta} = -\hat{\mathbf{z}}$, giving :

$$\mathbf{B}(r) = -\frac{\mu_0 m}{4\pi r^3}\hat{\mathbf{z}} = -\frac{\mu_0 I_s \pi a^2}{4\pi r^3}\hat{\mathbf{z}}. \tag{11}$$

with r the distance from the center (not the radius of any loop). Can we just integrate this over the surface S_p enclosed by the pan loop now? Unfortunately not – this is only valid for $r \gg a$, and much of the flux through the surface enclosed by the pan loop is at small radii (all of the return flux at $r < a$, for example). There is a trick, however: the total flux through the infinite plane $\mathbb{R}^2$ containing the current loop is zero (every field line crosses it twice, see the field in figure 5.55b fourth edition), so we can compute the flux for $r \geq b$ and take the negative:

$$\begin{aligned}
\Phi_p &= \int_{S_p} \mathbf{B}(r) \cdot d\mathbf{a} = -\int_{\mathbb{R}^2 - S_p} \mathbf{B}(r) \cdot d\mathbf{a} \\
&= -\int_b^\infty B_z(r) r dr d\phi = \frac{\mu_0 I_s \pi a^2}{2}\int_b^\infty \frac{r dr}{r^3} \\
&= \frac{\mu_0 I_s \pi a^2}{2b}.
\end{aligned}$$

From here we can use $\mathcal{E} = -\frac{d\Phi_p}{dt}$ as above to find the induced current.

Spinning cylinder [5 points]

An infinite cylinder of radius R carries a uniform surface charge σ. Although it has negligible mass, significant work is still needed to set it spinning around its axis.

11. (1 point) What opposes the acceleration of the cylinder? Answer in a few words (a sentence at most), but be specific.

Solution: Few-word answer: a Faraday/induced electric force/field/emf.

One-sentence answer: Changing the rotation of the cylinder changes a current, thus changing a magnetic field, which induces an opposing electric field.

Award no points for 'the electromagnetic force' (too vague, that covers basically everything in this course), 'a force', 'something', etc. Also award no points for a paragraph-length answer – the question clearly asks for a shorter answer.

Award half a point for 'the electric field'. This is in principle correct, but not sufficiently specific. The static (Coulomb) electric field of the charge density does not oppose the acceleration. (Only the induced (Faraday) electric field does.)

Also award half a point for 'electromagnetic induction' or 'Faraday's law' or 'Lenz's law'. Again, this is correct , but not sufficiently specific. No induced *current* need be involved here, for example: the cylinder can be an insulator carrying charge.

'Self-inductance' by itself is worth no points: that's a geometric quantity that does not directly oppose anything. Of course self-inductance can be involved in a correct explanation. If the explanation involving self-inductance is almost correct (e.g. changes in 'charge' instead of changes in 'current'), you earn half a point.

12. (4 points) Find the minimum required work, per unit length of the cylinder, to set it spinning at an angular velocity of ω radians/second. Hint: a spinning charged cylinder is equivalent to...

Solution: The surface current density is $\mathbf{K} = \sigma\mathbf{v} = \sigma\omega R\hat{\phi}$. (Or minus this, depending on which direction the cylinder spins.) [1pt]

The current in a spinning cylinder is the same as for a solenoid, so the magnetic field inside is

$$\mathbf{B} = \mu_0 K\hat{\mathbf{z}} = \mu_0\sigma\omega R\hat{\mathbf{z}}. \tag{12}$$

and outside it is zero. [1pt]

Finally, we find the energy in the magnetic field by integrating over a region of length L. This is easy because the field is uniform inside and zero outside:

$$\begin{aligned} W &= \frac{1}{2\mu_0}\int B^2 d\tau = \frac{\pi R^2 L}{2\mu_0}(\mu_0\sigma\omega R)^2 \quad \text{[1pt]} \\ W/L &= \frac{\pi}{2}\mu_0\sigma^2\omega^2 R^4 \end{aligned}$$

By conservation of energy, this is the minimum work required to set the cylinder spinning [1pt].

Scoring aid: If $W = \frac{1}{2\mu_0}\int B^2 d\tau$ is used in a somewhat reasonable argument, at least one point. If a completely wrong magnetic field was used (for example, that of an infinite wire), maximum 2 points for a completely correct answer (in this case, you will find ∞). If you use a trivial $\mathbf{B}$ (e.g. a constant), maximum 1 points due to a substantial simplification.

Common mistake: using $B = \mu_0 IN/L$ without expressing $IN/L = \sigma\omega R$. This results in

$$W = \frac{\pi R^2 L}{2\mu_0}\left(\frac{\mu_0 IN}{L}\right)^2 = \frac{\pi\mu_0 R^2 N^2 I^2}{2L} \tag{13}$$

so

$$W/L = \frac{\pi\mu_0 R^2 N^2 I^2}{2L^2} = \frac{\pi}{2}\mu_0 R^2 n^2 I^2 \tag{14}$$

Assuming everything else was done correctly (including drawing the analogy with the solenoid), this is worth a maximum of three points (since you only missed the first step).

Remark: If you remember only $B = \mu_0 nI$ as the field of a solenoid, with n the windings per unit length, how do you know to replace nI by $K = \sigma\omega R$? Notice nI is the current passing through an axial section (fixed θ) of a unit length of the solenoid – in other words, the surface current density K. (Which is what matters for $\mathbf{B}$; it's the enclosed current in the axial Amperian loop used to derive the field: see example 5.9.) If it's still unclear, try advancing time by one second. Then a point on the cylinder moves by ωR, so $\sigma\omega R$ of charge passes through the section (σ is charge per surface area, and we have unit length), so that must be the enclosed current in our Amperian loop.

Alternative: You may also have taken σ as the *total charge*. The question did not explicitly say 'density', and although surface charge density is a common meaning of the symbol σ, we will not penalize you for a strict reading of the question. In this interpretation, the answer will have $\sigma/A = \sigma/(\pi R^2 L)$ instead of σ, so an otherwise correct answer would be:

$$W/L = \frac{\pi}{2}\mu_0 \frac{\sigma^2}{(\pi R^2 L)^2}\omega^2 R^4 = \frac{\mu_0 \sigma^2 \omega^2}{2\pi L^2} \tag{15}$$

Only writing down 'it's equivalent to a solenoid' and nothing else: half a point.

Other common mistakes: Using the magnetic field of an infinite wire (?!), or a single loop. If you do this, you should be integrating to find W since the field is not homogeneous, but many people multiplied by A anyway. Many did use the solenoid field, but with an incorrect current; e.g. $I = \frac{\sigma\omega R}{2\pi R}$.

This concludes the test. When you are finished, please:

- Write your **name and student number on every sheet**!
- If you used two sheets, mark them 'Sheet 1/2' and 'Sheet 2/2'. When you hand them in, bind them with **two paperclips** on opposite sides.
- Feed your solutions to the wooden box. Not in the box = not graded.
- Return your formula sheet and *unused* paper. Take this question paper, your cheat sheet, and *used* scratch paper home.