

Electricity and Magnetism Test 5

23 May 2025, 18:30-20:30

- You may use your double-sided A4 cheat sheet, the provided formula sheet, and a calculator.
- Please leave margins for grading. Do *not* use this paper, or the white scratch paper, for final answers.
- Clearly indicate directions of vector quantities. Make or copy diagrams if it helps you.
- The maximum score is 30 points. Good luck!

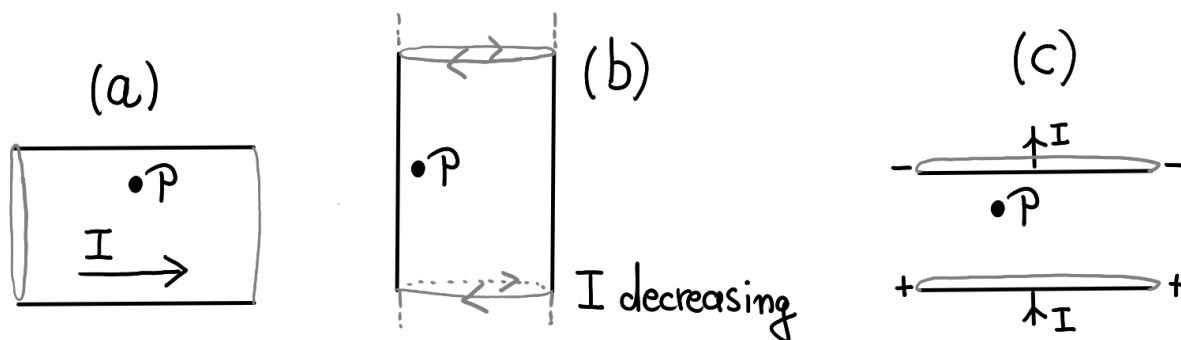
I. Short questions [12p]

1. Consider the function $f(x, t) = A \sin(kx) \cos(kvt)$.
 - (a) (2 points) Show that this is a solution to the one-dimensional wave equation with wave velocity v and distance x along the medium.
 - (b) (1 point) What type of wave does this specific solution represent? Just name it, no explanation needed.
2. Consider red and blue light in a non-magnetic linear medium whose permittivity increases with frequency.
 - (a) (1 point) State which color has higher frequency. No explanation needed.
 - (b) (2 points) Based on this, compare the **speed** (phase velocity) and **wavelength** of the two colors. Explain which color has the lower/higher value, or why there is no difference.
3. (3 points) Briefly define/explain the terms **plane of incidence**, **s-polarization**, and **Brewster's angle** (explain what it means, not how to calculate it).
4. (3 points) A traveling monochromatic electromagnetic plane wave in vacuum has the magnetic field

$$\tilde{\mathbf{B}}(\mathbf{r}, t) = -\tilde{B}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} \hat{\mathbf{y}} \quad (1)$$

with $\mathbf{k} = (-k, 0, k)/\sqrt{2} = k(\hat{\mathbf{z}} - \hat{\mathbf{x}})/\sqrt{2}$. Give the corresponding expression for the **electric** field $\tilde{\mathbf{E}}(\mathbf{r}, t)$ of this wave. Use only quantities that appear on the right-hand side of the equation above, fundamental constants, and standard unit vectors. Do not leave unevaluated cross products.

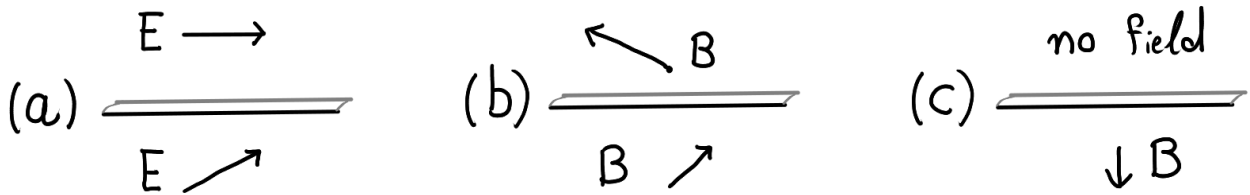
II. Pictures of fields [6p]



The three diagrams above show (a) a cylindrical ohmic wire with steady current flowing right, (b) a long solenoid with a *slowly decreasing* current directed clockwise when viewed from above, and (c) a broad parallel-plate capacitor that is charging slowly. The point P is always on a central cross-section of the object (in the dimension in/out of the paper), so it's inside the wire, solenoid, or capacitor, respectively.

5. (3 points) Draw or indicate the directions of the vectors $\mathbf{E}$, $\mathbf{B}$, and $\mathbf{S}$ at the point P in each problem. No explanations needed. (Don't just say 'x-direction', make clear what you mean. If a vector is zero, say so, don't omit it.)

Three (completely different) diagrams below show situations very close to a surface, which is viewed edge-on. Other sources of fields may exist far from the surface. Only consider clearly visible differences, not minute imperfections of the drawing.



6. (3 points) For each diagram, state whether it is possible or not. For *possible* diagrams, *clearly* indicate the direction/sign of the surface current/charge. For *impossible* diagrams, state which boundary condition is violated (either give the formula or name the Maxwell equation it derives from). No explanations needed.

III. Derivations [8 points]

7. (4 points) From the continuity equation for free charge, show that an initial free charge density $\rho_f(t=0)$ in an ohmic medium dissipates with a characteristic time ϵ/σ , where ϵ is the permittivity and σ the conductivity of the medium.
8. (4 points) Show that $u = \frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2$ satisfies the conservation of energy equation:

$$\frac{\partial u}{\partial t} = -\nabla \cdot \mathbf{S} - \mathbf{E} \cdot \mathbf{J}, \quad (5)$$

with $\mathbf{J}$ the current density and $\mathbf{S}$ the Poynting vector. Hint: use a product rule from your formula sheet.

IV. Levitations [4 points + 2 bonus]

A student suggests a lecture demonstration: levitate a tiny piece of aluminum foil with a laser pointer. Suppose the laser pointer has a power of 5.0 mW and an area of 1.0 mm². The local gravitational acceleration is $g = 9.81 \text{ m/s}^2$, and the density of aluminum is 2.71 g/cm³. Assume the foil is fully reflective and our piece has the same surface area as the laser beam.

9. (4 points) Compute the maximum thickness of a piece of aluminum foil that can be levitated by the radiation pressure of this laser. (Answer with a correct unit and number of significant figures.)

A final bonus question follows. This is challenging and rewards only a few points. Attempt it only if you have sufficient time.

Another student proposes to add a partially reflective non-absorbing mirror with reflection coefficient $R = 0.999$ between the laser and the reflective foil. The mirror reflects light from either direction, and the provided coefficient characterizes the mirror as a whole (not one of its interfaces). Assume that the distance between the foil and the mirror is an exact multiple of the laser wavelength, that the foil and mirror are both perpendicular to the laser beam, and that light returned to the laser pointer is lost.

10. (2 bonus points) By what factor is the radiation pressure on the foil multiplied, compared to the setup without the mirror?

This concludes the test. When you are finished, please:

- Write your **name and student number on every sheet!**
- If you used two sheets, mark them 'Sheet 1/2' and 'Sheet 2/2'. When you hand them in, bind them with **two paperclips** on opposite sides.
- Feed your solutions to the wooden box. Not in the box = not graded.
- Return your formula sheet and *unused* paper. Take this question paper, your cheat sheet, and *used* scratch paper home.