Electricity and Magnetism Test 6

16 June 2025, 8:30 – 10:30

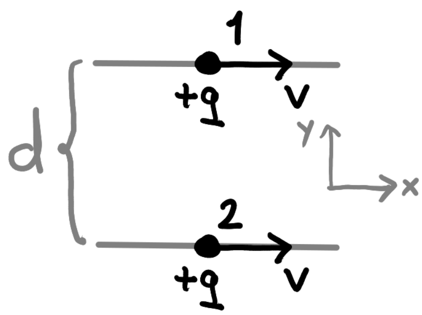
- You may use your double-sided A4 cheat sheet, the provided formula sheet, and a calculator.
- Please leave margins for grading. Do *not* use this paper, or the white scratch paper, for final answers.
- Clearly indicate directions of vector quantities. Make or copy diagrams if it helps you.
- The maximum score is 32 points. Good luck!

I. Short questions [13p]

1. Consider two events with $\Delta x^\mu \Delta x_\mu > 0$ (under the sign conventions used throughout this course).
   (a) (1 point) What does $\Delta x^\mu \Delta x_\mu$ represent in general?
   (b) (2 points) $\sqrt{\Delta x^\mu \Delta x_\mu}$ for these two events is a specific physical quantity measured by a specific observer. Give the name of this quantity, and specify what observer it is measured by.
2. A charged particle is moving in a circular orbit with constant radius and constant speed.
   (a) (2 points) Briefly explain why an external source of energy is needed to keep the particle in this orbit. You can quote results derived in class/Griffiths without derivation.
   (b) (1 point) Suppose the circle is large enough that the center can be considered far from the particle for the purpose of calculating the fields. Can we detect electromagnetic waves at the point at the center of the circular orbit? (Explain very briefly, one sentence suffices.)
3. (3 points) You measure a magnetic field strength of 1.0 Tesla at some point P in space. Your friend moving at $v = 0.60c$ relative to you measures no *electric* field at P. Find the minimum and maximum possible magnitude in V/m of the electric field that you measure at P.
4. (4 points) Show that the Maxwell-Ampere law $\mu_0 \mathbf{J} = \nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$ becomes $\mu_0 \mathbf{J} = -\Box^2 \mathbf{A}$ in Lorenz gauge.

II. Two point charges [7pt]

Two point charges, each of charge $+q$, move in parallel with speed v (on straight rails) in the $+\hat{\mathbf{x}}$ direction. The rails are a distance d apart in the $+\hat{\mathbf{y}}$ direction, with charge number 1 on top. See the figure on the right for illustration. (Do *not* assume $v \ll c$ in this question!)



5. (1 point) How far apart are the charges in the frame where they are at rest?
6. (6 points) Find the force on charge 1 due to the fields of charge 2, in the frame specified and drawn in the question. Start from the frame where the charges are at rest, then use the field transformation rules.

III. Rotated dipole [6p]

You are at a distance $41\pi c/\omega$ in the $+\hat{\mathbf{y}}$ direction from a tiny, forever-oscillating dipole with $\mathbf{p}(t) = p_0 \cos(\omega t)\hat{\mathbf{x}}$ with $p_0 > 0$. The 'radiation zone' is the region of space where the usual dipole radiation solution we derived in class is accurate.

7. (1 point) Briefly justify why you are in the radiation zone. [1p]
8. (5 points) Determine the *direction* in Cartesian coordinates of the electric field at $t = 0$ at your position. If you rotate or change coordinates, make clear how you do this (perhaps with a drawing).

IV. Can you gauge this? [6p]

Given some vector potential $\mathbf{A}(\mathbf{r}, t)$ and scalar potential $V(\mathbf{r}, t)$, you perform transformations:

$$\mathbf{A}' = \mathbf{A} + \alpha_0 \mathbf{r} \exp\left(-\frac{r^2}{2a^2}\right) \sin \omega t, \tag{2}$$

$$V' = V + \beta_0 \exp\left(-\frac{r^2}{2a^2}\right) \cos \omega t. \tag{3}$$

where a, α_0, β_0 and ω are constants.

9. (6 points) Find the condition under which this is a gauge transformation. Your answer will be an equation involving a, α_0, β_0 and ω.

This concludes the test. When you are finished, please:

- Write your **name and student number on every sheet**!
- If you used two sheets, mark them 'Sheet 1/2' and 'Sheet 2/2'. When you hand them in, bind them with **two paperclips** on opposite sides.
- Feed your solutions to the wooden box. Not in the box = not graded.
- Return your formula sheet and *unused* paper. Take this question paper, your cheat sheet, and *used* scratch paper home.